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Approximate models for the probability distributions for inventory position and net inventory for Navy repairable items

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THESIS

APPROXIMATE MODELS FOR THE PROBABILITY DISTRIBUTIONS FOR INVENTORY POSITION AND NET INVENTORY FOR NAVY REPAIRABLE ITEMS

by

Stephen Baker

September, 1994

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Approximate Models for the Probability Distributions
for Inventory Postion and Net Inventory
for Navy Repairable Items

by

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ABSTRACT

This thesis represents the most recent step in the development of a new wholeslale level repairable item inventory model for the Navy. Distinguishing features of the process by which repairable items are managed in the Navy are batching of both repair and procurement orders and a specific repair assessment policy which sees repairable items enter repair agencies in batches but leave as single items. Approximate steady state probability distributions for inventory position and net inventory are developed and these are extensively tested against simulation results. Graphical analysis and testing using nonparametric methods suggest that the approximations developed closely resemble the distributions obtained via simulation.

TABLE OF CONTENTS

EXECUTIVE SUMMARY

A. PURPOSE

The purpose of this thesis is to develop approximately correct probability distributions which describe the steady state performance of a Navy repairable item inventory. Specifically, the distributions of inventory position and net inventory are of interest. Once developed these distributions can be used in an inventory management mathematical model for repairable items.

B. BACKGROUND

Since late 1986 work has been undertaken at the Naval Postgraduate School to develop a new mathematical model for wholesale level repairable item inventory control. The first step in the development process is to determine probability distributions which describe the steady state behavior of a repairable item inventory. The complex mix of stochastic processes which occur in the life of a repairable item has discouraged previous analytical approaches. This has led to the application of simulation techniques to determine approximately correct distributions.

C. RESULTS

This thesis has shown that it is possible to incorporate an analytical approach in the development of approximate probability distributions for inventory position and net inventory. Using established stochastic modeling results and the previously derived approximation for inventory position, approximations for the mean and variance of net inventory were developed. Two approximating distributions for net inventory were proposed and their performance along with that of the inventory position approximation were tested against results obtained through simulation.

The results of this thesis support previous research results that the convolution of two discrete Uniform random variables provides a robust approximation for the steady state distribution of inventory position.

The approximations developed in this thesis for the mean of net inventory closely matched simulation output, whereas the variance estimates consistently overestimated the simulation results. Despite these varied outcomes, the Normal and convolution approximations proposed for the steady state distribution of net inventory appeared to closely resemble the simulation output. On the basis of the graphical analysis and testing using nonparametric methods the convolution approximation seems a robust approximation for net inventory over a wide range of system inputs.

D. RECOMMENDATIONS

This thesis represents a significant step in the development of a repairable item inventory control model for the Navy. The next logical step is for the approximations developed in this thesis to be compared against observed inventory performance. Subject to a favorable outcome to this comparison a readiness-based repairable item inventory model using the approximations developed in this thesis should be readily forthcoming.

I. INTRODUCTION

A. BACKGROUND

Significant changes in wholesale supply management responsibility have been implemented throughout the Department of Defense and the Services since the 1960's. One of the most recent changes has come with the Defense Management Review Decision 926, "Consumable Item Transfer", which included the transferring of certain stock management responsibilities from the Navy to the Defense Logistics Agency (DLA). One effect of such changes has been to alter the make-up of wholesale level secondary support item [Ref. 1] inventories maintained by the Navy.

Secondary support items used in the Navy are broadly classified as either consumable or repairable. In general, consumable items are consumed in use and disposed of at the time of failure. On the other hand, repairable items can be made to function by a repair process after they fail.

In the past the Navy has managed significant wholesale level inventories of both consumable and repairable items. However, the changes in supply management responsibilities mentioned above have resulted in most consumable items being transferred to DLA. Thus, the Navy's primary focus for wholesale level inventory management is now repairable items.

The differing nature of repairable and consumable items dictates that there will be differences in the way that the respective inventories are managed. The Navy has separate mathematical models which aim to determine optimal strategies for managing wholesale level inventories of consumable and repairable items. However, the current repairable item model, which was developed initially in the late 1960's and modified in 1984, is not supported by any rigorous mathematical development and remains founded upon a minor variation of the model used to manage consumable items. It is not surprising then that the repairable item model displays a low level of efficacy.

Since late 1986 work has been undertaken at the Naval Postgraduate School to develop a new mathematical model for wholesale level inventory management of repairable items. Early attempts to develop an analytical model were unsuccessful because of the complex mix of stochastic processes which occur in the life of a repairable item. As a result, the focus of recent work has shifted to using simulation techniques to study the process.

The objective of such efforts is to determine approximately correct theoretical probability distributions which describe the steady state behavior of a repairable item inventory. These distributions then can be used in an inventory management mathematical model for repairable items.

B. PRIOR ANALYSIS

The thesis completed by Maher [Ref. 2] is the most recent step in developing an inventory control model for Navy repairable items using simulation techniques. While previous simulations had been developed, Maher's work remains significant because:

- the simulation model incorporated refinements that more accurately describe how wholesale level repairable item inventories are maintained in the Navy;
- some progress was made in the verification of a previously derived probability distribution for inventory position, and in the identification of a probability distribution for net inventory;
- in the specific context of the Navy repairable item inventory, a definition of safety stock was proposed.

In reviewing the general inventory model literature with respect to this thesis four system descriptors are significant:

- Batch Ordering policies - which prescribe the quantity of a repairable item to be procured or repaired.
- Repair Assessment/Inspection policies - which prescribe the process by which repairable items are initially assessed and prepared for repair at a repair center.
- Shipment Policies - which prescribe the process by which repaired and procured items are shipped from a supplier to a customer.
- Condemnation - being the process by which failed items are assessed as not being capable of repair.

As will be described in this thesis, the Navy has specific requirements for each of these system descriptors in the management of repairable items. These requirements introduce complications which are not addressed by the current Navy repairable item model or by other prominent repairable item inventory models such as METRIC [Ref. 3]. This is not to say that there have not been inventory models proposed which deal with these more difficult issues. Indeed, the literature has examples of batch ordering and/or repair assessment models [Ref. 4,5,6, 7 and 8] which, in the main, apply approximating techniques to determine optimal inventory control strategies. However, the detailed nature of the repairables process does not lend itself to the adaptation of such models to the specific requirements of the Navy. For example, none of the models in the literature consider both repair and procurement order batching, or repair induction processes which see failed carcasses entering repair agencies as batches but leaving as single items. As a consequence, the Navy's repairables process has been subject to individual and detailed analysis at the Naval Postgraduate School.

C. OBJECTIVES

This thesis follows directly on from Maher's work with the specific objectives to:

- use simulation output to study the probability distributions of net inventory and inventory position for a Navy wholesale level repairable item inventory,
- determine the functional relationship between inputs to the repairable item management system and parameters of these distributions,
- develop approximately correct theoretical probability distributions for net inventory and inventory position.

D. SCOPE

Maher simulated a Navy repairable item inventory for a specific set of system inputs and employed regression analysis techniques to conduct preliminary investigations into functional relationships between these inputs and distributional parameters. In contrast, this thesis conducts the same simulation but over a greater input range, and uses stochastic modeling techniques to derive approximations to distributional parameters.

E. PREVIEW

Chapter II provides a plain language system description of the processes which work together to determine the wholesale levels of Navy repairable items at any point in time. Chapters III and IV develop stochastic models for repairable item wholesale inventory levels to support approximations for the steady state distribution of inventory position and net inventory. Chapter V contains the comparison and analysis of results obtained via simulation and stochastic modeling. Finally, Chapter VI summarizes the thesis research and findings, offers conclusions, and makes recommendations for further analysis.

II. CONCEPTUAL MODEL

A plain language system description is a key ingredient of any study which seeks to understand the relationships between various system components and/or to predict system performance. In the case of this thesis, such a description forms the basis for both the simulation of the repairable system and the analysis of its stochastic features. Hereafter this description shall be referred to as the "conceptual model".

The conceptual model for the management of wholesale level inventories of repairable items in the Navy assumes that there is a maximum inventory level, SW, and actual inventory levels vary from this maximum as a result of demands being placed by customers and two distinct but interrelated replenishment processes:

- the repair process
- the procurement process

To outline the conceptual model the movement of a repairable item between a ship (representing the ultimate user of a repairable item), inventory manager, wholesale stock, repair agency, and supplier will be described. A summary of the system inputs and variables used in the description and analysis of the conceptual model is contained in Appendix A.

The conceptual model assumes that demands for repairable items occur at a rate of one unit per requisition and that demand follows a Poisson distribution with a mean rate of D units per quarter. On receipt of a demand for a repairable item the inventory manager will satisfy the demand from wholesale stock on a "first come first served" basis or place the demand on backorder. The replenishment of wholesale stock levels following these demands will then be undertaken by the repair and procurement processes.

A. THE REPAIR PROCESS

When a repairable item fails onboard, the ship demands a replacement item and dispatches the failed item back to the inventory manager so that repair can be arranged. The beginning of the repair process for an individual repairable item is marked by the return of a failed repairable item to the inventory manager. The conceptual model assumes that when this return happens it is completed instantaneously. However, it is possible that the demand for a replacement item is not accompanied by the successful return of a failed repairable item to the inventory manager (for example, due to loss of the item in transit between the ship and the inventory manager). Therefore, the probability that any one demand will be accompanied by a failed item, or carcass, is called the Carcass Return Rate (CRR).

The inventory manager accumulates carcasses until a set number (Q_R) is reached in the repair batching process. At this time Q_R failed items are dispatched as a single batch to any one of an infinite number of repair agencies. Each repair agency is assumed to have infinite capacity to undertake repair. Not all carcasses in the batch will be capable of repair and thus initially the repair agency sequentially assesses and, if necessary, prepares the items for repair. The actual decision as to whether a carcass is repairable or not takes zero time, whereas the preparation for repair, described below, takes a fixed length of time. The probability that any one carcass randomly selected will be capable of repair is called the Repair Survival Rate (RSR). The first carcass is assessed at the moment the batch reaches the repair agency. At time t, when a carcass is assessed for repair, one of two outcomes will occur.

If the carcass is assessed as being capable of repair then a repaired item will be returned to the wholesale stock at time t+RTAT, where RTAT is the known and fixed time interval referred to as the Repair Turn Around Time (RTAT). The repair assessment of the next item in the batch (should there be one) will then occur at time t+REP, where REP is a known and fixed time interval. If the carcass is assessed as not being capable of repair then the inventory manager is immediately notified, the quantity in repair reduced by one, and the "attrition" is recorded in the procurement process at time t. The repair assessment of the next item in the batch (should there be one) will then occur instantaneously at time t. Therefore the time element REP can be thought of as the repair preparation time or the time delay in commencing actual repair caused by the need to conduct diagnostic tests, fault isolation procedures and material preparation etc for an individual repairable item prior to considering the next item in the batch. On the basis of the repair process described above REP is necessarily less than or equal to RTAT.

B. THE PROCUREMENT PROCESS

When a ship demands a replacement for a failed repairable item but, for whatever reason, the failed item is not returned to the inventory manager, or an item is assessed in the repair process as not being capable of repair then an attrition is said to have occurred. Attritions require the procurement of additional items to maintain inventory levels and, thus, an attrition marks the start of the procurement process. As with the repair process, the inventory manager accumulates attritions until a set number (Q_P) is reached in the procurement batching process. At this time the inventory manager places an order of size Q_P with a supplier which arrives in wholesale stock (as a single batch) a fixed time interval later. This time interval is referred to as the Procurement Lead Time (PCLT) and is assumed to be significantly greater than RTAT.

Figure 1 shows the movement of repaired and procured items (RFI - Ready for Issue), carcasses and attritions in the repair and procurement processes.

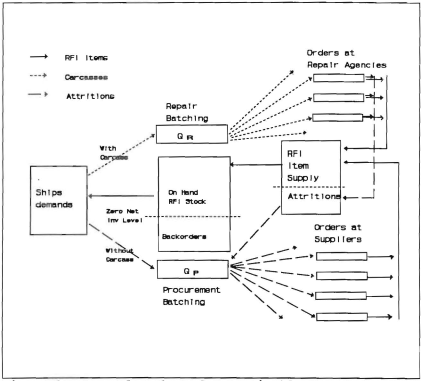


Figure 1. Item Flow Chart for Repairable Item Conceptual Model.

C. MAGNITUDE OF MODEL INPUTS

Based on the conceptual model described, there are eight system inputs which will determine the distribution of inventory levels at any particular point in time.

The range of system inputs that have been considered in this thesis together with the basis for their selection are outlined as follows:

- Quarterly Demand Rate (D): In the case of repairable items the quarterly demand rate tends to be low. For example, on the basis of sample information provided by Mr. John Boyarski at the Ships Parts Control Center, demand rates less than 9 units per quarter tend to dominate. Rates extending to 40 units per quarter are, however, considered.
- Carcass Return Rate (CRR): Financial redemption values applied to repairable items in the Navy provide incentives for users to return failed carcasses to the inventory manager. Return rates in the range from 0.4 to 1.0 are considered.
- Repair Survival Rate (RSR): Repairing a failed item consumes considerable effort and resources. As a consequence, this would dictate that failed carcasses have a reasonable probability of being repaired. Repair survival rates in the range from 0.4 to 1.0 are considered.
- REP: At the component and sub-assembly levels it seems reasonable to expect that this time element might be reasonably short, perhaps not exceeding two weeks. In more complex sub-assembly and assembly levels this delay due to diagnostic testing and fault isolation, etc., may be significant. REP delays in the range from 0.0 to 1.6 quarters are considered.
- Repair Turn Around Time (RTAT): Historical evidence shows that RTAT varies between organic and commercial repair agencies. On the basis of data provided by Mr. John Boyarski turnaround times in the range from 0.3 to 2.3 quarters are considered.

- Procurement Lead Time (PCLT): Repairable items generally experience long resupply times. On the basis of information provided by Mr. Boyarski lead times in the range from 3.5 to 8.5 quarters are considered.

- Batch/Order Quantities (Q_R/Q_P): While classified as inputs for the purposes of this thesis, in any inventory model these quantities will become decision variables. Therefore, in testing distributional approximations these inputs should remain reasonably unconstrained. However, there are some practical relationships that might be reasonably expected among Q_R, D, REP and RTAT. Repairable items that experience low demand rates and/or large RTAT/REP values are likely to be batched for repair in small quantities. The converse is also true. Therefore Batch/Order quantities up to 42 units which apply these relationships are considered.

- Maximum Inventory Level (SW): The nature of the conceptual model is such that any value of SW would serve equally well for testing purposes. To provide continuity the same value as used by Maher, 72 units, is used throughout this thesis.

The stochastic modeling conducted in this thesis also assumes that there is some level of material flow through the procurement and repair processes. This assumption translates to the joint condition that CRR>0 and product CRR*RSR<1. The case when CRR equals zero represents the strict procurement model, the Navy application of which was discussed by Maher [Ref. 2:pp. 6-18]. The case when CRR*RSR equals one represents the strict repair model. While this later situation is highly unlikely to arise, the arguments developed in this thesis can be easily extended to cover such a case.

D. MEASURES OF INTEREST

The inventory level measures which are of primary interest to this thesis are Inventory Position (IP) and Net Inventory (NI) and are defined as follows:

IP = On Hand RFI Stock + On Order + In Repair - Backorders,

which, in the context of the conceptual model, is equivalent to

IP = SW - Carcasses and attritions currently being accumulated by the Inventory Manager.

NI = On Hand RFI Stock - Backorders,

which is equivalent to,

NI = IP - On Order - In Repair.

Here RFI refers to the items which are "as new" and Ready for Issue.

III. INSTANTANEOUS REPAIR ASSESSMENT

The case of instantaneous repair assessment is when the time element REP in the conceptual model is considered insignificant and set to zero. Under this condition, as a repair batch of size Q_R reaches a repair agency, the entire batch will be assessed instantaneously. Thus, all carcasses commence repair immediately and attritions are processed immediately. This case simplifies to some degree the stochastic analysis of the conceptual model, while at the same time providing a foundation for the more difficult scenario of non-instantaneous repair assessment.

The first section of this chapter reviews the development of the steady state distribution of inventory position. The second section considers the net inventory level for the case of instantaneous repair assessment, with key results being an approximation for the mean and variance, leading to a suggested probability distribution for net inventory.

A. STEADY STATE DISTRIBUTION OF INVENTORY POSITION

Maher [Ref. 2:pp. 26-32] outlined the development of the steady state distribution of inventory position for the repairables model as previously derived by Professor McMasters at the Naval Postgraduate School. To remain consistent with the notation introduced in the conceptual model, and for the purposes of clarity, the supporting arguments and result shall be summarized in this thesis.

To introduce notation, let

$I(t)$ = the inventory position at time t;

$B_R(t)$ = the number of carcasses in the repair batching process at time t; and

$B_P(t)$ = the number of attritions in the procurement batching process at time t.

Then, it follows from the conceptual model that

$$I(t) = SW - B_R(t) - B_P(t). \tag{1}$$

As demands are generated and accompanied by carcasses, $B_R(t)$ will increase by one until Q_R items are held in the repair batching process. At this later time a repair order will be generated instantaneously decreasing the number in the repair batching process to zero. It follows from the conceptual model that this demand will occur as a Poisson process, and therefore the repair batching process can be modeled as a continuous time Markov chain with state space$\{0,1,2 \ \ldots \ Q_R-1\}$ and a doubly stochastic transition matrix [Ref 9:p. 191]. Thus, the steady state distribution for the number of carcasses batching is given by

$$P(B_R(t)=w) = \frac{1}{Q_R} \qquad \textit{for } w = 0,1,2,\ldots Q_R-1. \tag{2}$$

Similarly, as attritions are generated, $B_P(t)$ will increase by one until Q_P items are held in the procurement batching process. At this later time a procurement order will be generated instantaneously decreasing the number in the procurement batching process to zero. As stated in the conceptual model, these attritions result both from demands not being accompanied by a carcass and carcasses being assessed beyond repair in the repair process. It follows from the conceptual model that this first source of attritions will form a Poisson process and occur at a constant rate. Results from Maher's work support the assumption that the second source of attrition can be approximated as a Poisson process [Ref 2.pp. 29-30]. Thus, the procurement batching process can also be modeled as a continuous time Markov chain with state space $\{0,1,2 \ldots Q_P\text{-}1\}$ and a doubly stochastic transition matrix. The steady state distribution for the number of attritions batching is approximated by

$$P(B_P(t)=v) \approx \frac{1}{Q_P} \qquad for\ v = 0,1,2,\ldots Q_P-1. \qquad (3)$$

There is no intuitive reason why at a particular point in time the two batching processes would be dependent and, therefore, the steady state probability distribution for inventory position can be approximated from the convolution of $P(B_R(t))$ and $P(B_P(t))$. The resulting distribution for $P(I(t))$ is shown in equation (4).

$$P[I(t)=SW-x] \approx \begin{cases} \dfrac{x+1}{Q_P Q_R} & \text{for } 0 \le x \le x_1 \\ \dfrac{Min(Q_P, Q_R)}{Q_P Q_R} & \text{for } x_1 < x \le x_2 \\ \dfrac{x_{Max}+1-x}{Q_P Q_R} & \text{for } x_2 < x \le x_{Max} \\ 0 & \text{otherwise;} \end{cases} \tag{4}$$

where x_{Max}, x_1, and x_2 are defined as

$$\begin{aligned} x_{Max} &= Q_P+Q_R-2; \\ x_1 &= Min(Q_P, Q_R)-1; \\ x_2 &= x_{Max}-x_1. \end{aligned} \tag{5}$$

Given that inventory position is approximated utilizing the convolution of two discrete Uniform random variables, it follows that

$$E[I(t)] \approx SW - \frac{Q_R-1}{2} - \frac{Q_P-1}{2}; \tag{6}$$

$$Var[I(t)] \approx \frac{(Q_R-1)(Q_R+1)}{12} + \frac{(Q_P-1)(Q_P+1)}{12}. \tag{7}$$

B. STEADY STATE DISTRIBUTION OF NET INVENTORY

To consider the distribution of net inventory let

$N(t)$ = net inventory level at time t;

$D(t_1,t_2)$ = number of demands in the period (t_1,t_2);

$D'(t_1,t_2)$ = number of demands in the period (t_1,t_2) accompanied by a carcass;

$D''(t_1,t_2)$ = number of demands in the period (t_1,t_2) not accompanied by a carcass; and

$R_{IR}(t_1,t_2)$ = number of carcasses inducted into repair agencies in the period (t_1,t_2) that will be assessed as repairable.

Since the time any item spends at a repair agency (which is at most RTAT) is strictly less than PCLT, it follows that in the time period (t-PCLT,t)

- Any quantity in procurement or actual repair at time t-PCLT will have been received by time t.
- Any repairable carcasses commencing actual repair in the period (t-PCLT,t-RTAT) will have been received by time t.

With instantaneous repair assessment, for any repairable carcass to commence actual repair in the time period (t-PCLT,t-RTAT) it must have been inducted into a repair agency in the time period (t-PCLT,t-RTAT). Thus it follows that

$$N(t) = I(t\text{-}PCLT) - D(t\text{-}PCLT,t) + R_{IR}(t\text{-}PCLT,t\text{-}RTAT), \quad (8)$$

which, after substituting equation (1) for I(t-PCLT), and noting that the Poisson random variable $D(t_1,t_2)$ can be decomposed into the sum of $D'(t_1,t_2)$ and $D''(t_1,t_2)$, becomes

$$N(t) = SW - B_R(t\text{-}PCLT) + R_{IR}(t\text{-}PCLT,t\text{-}RTAT) - D'(t\text{-}PCLT,t) - B_P(t\text{-}PCLT) - D''(t\text{-}PCLT,t). \quad (9)$$

1. Repairable Items (R_{IR}) Inducted Into Repair Agencies

As carcasses enter the repair process they are initially batched until a quantity Q_R is reached, at which point a repair order is instantaneously placed. From the conceptual model it is known that the inter-arrival times of these carcasses into the repair process are independent and identically distributed Exponential random variables with parameter (D*CRR). Thus, the times between the dispatching of successive repair batches of size Q_R to a repair agency will be independent and identically distributed Erlang random variables with parameters (Q_R,D*CRR). Let

T_{RB} = the time between the dispatching of successive repair batches of size Q_R to a repair agency;

and so

$$E[T_{RB}] = \frac{Q_R}{D\ CRR}; \tag{10}$$

$$Var[T_{RB}] = \frac{Q_R}{(D\ CRR)^2}. \tag{11}$$

In addition, let

$O_I(t)$ = the number of repair batches of size Q_R inducted into repair agencies in the time interval [0,t).

The counting process {$O_I(t)$, t>0} is a Renewal process as a consequence of the independent and identically distributed times between dispatching of successive repair batches.

In addition, from elementary Renewal theory the mean value function (i.e., the expected number of repair batches of size Q_R sent to repair agencies in the time interval (t_1,t_2)) can be approximated using Blackwell's theorem [Ref 10:p.41] as

$$E[O_I(t_1,t_2)] \approx \frac{(t_2-t_1)}{E[T_{RB}]} = \frac{D\ CRR\ (t_2-t_1)}{Q_R}, \tag{12}$$

where

$O_I(t_1,t_2)$ = the number of repair batches of size Q_R inducted into repair agencies in the time period (t_1,t_2).

Furthermore, if

w_i = the number of repairable carcasses in the ith repair order of size Q_R,

then w_i are independent and identically distributed Binomial random variables with mean and variance defined by

$$E[w_i] = Q_R\ RSR; \tag{13}$$

$$Var[w_i] = Q_R\ RSR\ (1-RSR). \tag{14}$$

Therefore the counting process $\{R_{IR}(0,t),\ t>0\}$ can be classified as a Renewal Reward Process [Ref 10:p. 310] or Cumulative Process [Ref 11:p. 91] described by

$$R_{IR}(t_1,t_2) = \begin{cases} \sum_{i=1}^{O_I(t_1,t_2)} w_i & \text{for } O_I(t_1,t_2) = 1,2,\ldots \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

The approximate expected number of items inducted into repair agencies in the period (t_1,t_2) that will be assessed as repairable follows as

$$\begin{aligned} E[R_{IR}(t_1,t_2)] &= E[\ E[R_{IR}(t_1,t_2)/O_I(t_1,t_2)]\ ] \\ &= E[O_I(t_1,t_2)\ Q_R\ RSR] \\ &= Q_R\ RSR\ E[O_I(t_1,t_2)] \\ &\approx D\ CRR\ RSR\ (t_2-t_1)\ . \end{aligned} \tag{16}$$

Conditioning can also be applied to approximate the variance of $R_{IR}(t_1,t_2)$. Relaxing the time interval notation, then

$$\begin{aligned} Var[R_{IR}] &= E[Var[R_{IR}/O_I]] + Var[E[R_{IR}/O_I]] \\ &= E[O_I\ Q_R\ RSR\ (1-RSR)] + Var[O_I\ Q_R\ RSR] \\ &= Q_R\ RSR\ (1-RSR)\ E[O_I] + Q_R^2\ RSR^2\ Var[O_I]\ ; \end{aligned} \tag{17}$$

where the variance for the Renewal Process $\{O_I(t),\ t>0\}$ is approximated by [Ref 11:p.70]

$$\begin{aligned} Var[O_I(t_1,t_2)] &\approx \frac{Var[T_{RB}]\ (t_2-t_1)}{E[T_{RB}]^3} \\ &\approx \frac{D\ CRR\ (t_2-t_1)}{Q_R^2}\ . \end{aligned} \tag{18}$$

Substitution of equations (12) and (18) into (17) leads to

$$Var[R_{IR}(t_1,t_2)] \approx D\ CRR\ RSR\ (t_2-t_1)\ . \tag{19}$$

A further result of interest is the covariance between O_I and R_{IR}, which follows from Cox [Ref 11:p.94] as

$$\begin{aligned} Cov[R_{IR}, O_I] &\approx Var[O_I]\ Q_R\ RSR \\ &\approx \frac{D\ CRR\ RSR\ (t_2 - t_1)}{Q_R}. \end{aligned} \tag{20}$$

2. Expected Net Inventory

The preceding results can be used to determine the expected net inventory. Specifically, taking the expected value of equation (9) and relaxing the time interval notation,

$$\begin{aligned} E[N(t)] &= SW - E[B_R] + E[R_{IR}] - E[D'] \\ &\quad - E[B_P] - E[D'']. \end{aligned} \tag{21}$$

Using the results of equations (6) and (16) leads to

$$\begin{aligned} E[N(t)] &\approx SW - \frac{Q_R - 1}{2} + D\ CRR\ RSR(-RTAT + PCLT) \\ &\quad - D\ CRR\ PCLT - \frac{Q_P - 1}{2} - D(1 - CRR)\ PCLT; \end{aligned} \tag{22}$$

which simplifies to

$$\begin{aligned} E[N(t)] &\approx SW - \frac{Q_R - 1}{2} - \frac{Q_P - 1}{2} \\ &\quad - D\ CRR\ RSR\ RTAT \\ &\quad - D(1 - CRR\ RSR)\ PCLT. \end{aligned} \tag{23}$$

By way of validation, consider the situation where Q_R and Q_P are both set equal to one, and REP is set equal to zero. In this situation a carcass arriving at the repair process at time t is immediately sent to a repair agency to undergo instantaneous assessment and, should it be an attrition, it will enter the procurement process at the same time t. Thus, with infinite repair and supply capacity this situation is the same as operating two independent infinite server queues with Poisson arrivals and fixed service times. Specifically, this situation will be characterized by

- An infinite server queue in the repair process experiencing Poisson demand arrivals with rate D*CRR*RSR and fixed service time of RTAT;
- An infinite server queue in the procurement process experiencing Poisson demand arrivals with rate D*(1-CRR*RSR) and fixed service time of PCLT.

From Queuing Theory [Ref 9:pp. 225-226] it is known that the expected net inventory level in this case will be

$$E[N(t)] = SW - D\ CRR\ RSR\ RTAT - D\ (1 - CRR\ RSR)\ PCLT. \quad (24)$$

The same result follows from substitution of Q_R and Q_P equal to one in equation (23).

3. Variance of Net Inventory

To consider the random variable dependencies in the case of instantaneous repair assessment a convenient way to rewrite equation (9) is

$$N(t) = SW - R(t) - R'(t) - P(t); \qquad \textbf{(25)}$$

where

$$\begin{aligned} R(t) &= B_R(t\text{-}PCLT) - R_{IR}(t\text{-}PCLT, t\text{-}RTAT) + D'(t\text{-}PCLT, t\text{-}RTAT); \\ R'(t) &= D'(t\text{-}RTAT, t); \\ P(t) &= B_P(t\text{-}PCLT) + D''(t\text{-}PCLT, t). \end{aligned}$$

(26)

The independence of the repair batching process (B_R) and the procurement batching processes (B_P) underlies the development of the steady state distribution of inventory position. In addition, established results for the Poisson process dictate that D" and D' are independent, being Poisson processes formed from the categorization of the Poisson process D. Finally, R_{IR}(t-PCLT,t-RTAT) and B_R(t-PCLT) are formed from demand type D' which occurs before time t-RTAT and t-PCLT, respectively. Thus, being functions of independent random variables, it follows that R(t), R'(t) and P(t) are independent. Furthermore, the number of attritions in the procurement batching process at a specific point in time is independent of the demand which follows in a subsequent period. Therefore, B_P(t-PCLT) and D"(t-PCLT,t) are independent. Based on these facts

$$Var[N(t)] = Var[R(t)] + Var[R'(t)] + Var[P(t)]; \qquad (27)$$

where

$$\begin{aligned} Var[P(t)] &\approx \frac{(Q_P - 1)(Q_P + 1)}{12} + D(1-CRR)\,PCLT; \\ Var[R'(t)] &= D\ CRR\ RTAT. \end{aligned} \qquad (28)$$

To examine the variance of R(t), consider the repair batching process at time t-PCLT. This process will continue to accumulate carcasses until a total of Q_R is held. In this case the time elapsed subsequent to t-PCLT will be equivalent to the length of time from t-PCLT until the next renewal in the Renewal Process formed by the generation of repair orders. In the situation when this remaining batching time is large in comparison to the time period (t-PCLT,t-RTAT), any repairable item inducted into a repair agency in the period (t-PCLT,t-RTAT) is highly likely to have been in the repair batching process at t-PCLT. Thus, under these circumstances a significant degree of dependency would be expected between B_R(t-PCLT) and R_{IR}(t-PCLT,t-RTAT). An alternative situation is when the remaining batching time is small in comparison to PCLT-RTAT. In this case any repairable item inducted into a repair agency in the time period (t-PCLT,t-RTAT) is more likely to be as a result of demand D' subsequent to t-PCLT than to have been in the batching process at t-PCLT. These two cases illustrate the expected dependencies between B_R(t-PCLT), R_{IR}(t-PCLT,t-RTAT) and D′(t-PCLT,t) under instantaneous repair assessment. Additionally, based on these arguments the covariance between B_R(t-PCLT) and R_{IR}(t-PCLT,t-RTAT) is expected to be non-negative.

Thus, relaxing time interval notation, it follows that the variance of R(t) is

$$\begin{aligned} Var[R] &= Var[B_R] + Var[R_{IR}] + Var[D'] \\ &\quad - 2\ Cov[B_R, R_{IR}] - 2\ Cov[R_{IR}, D'] . \end{aligned} \tag{29}$$

To determine the covariance between R_{IR} and D′ consider that in any fixed time interval (t-PCLT,t-RTAT)

$$D'(t\text{-}PCLT, t\text{-}RTAT) = Q_R\ O_I(t\text{-}PCLT, t\text{-}RTAT) + \alpha; \tag{30}$$

where α is a random variable in the range

$$\begin{aligned} 0 \le \alpha \le Q_R - 1 &\qquad for\ \ O_I = 0; \\ -(Q_R-1) \le \alpha \le (Q_R-1) &\qquad for\ \ O_I \ge 1. \end{aligned} \tag{31}$$

Thus

$$Cov[R_{IR}, D'] = Cov[R_{IR}, Q_R O_I + \alpha] . \tag{32}$$

In a probabilistic sense, the number of repairable carcasses entering repair agencies, R_{IR}, is a function of the number of repair batches of size Q_R accumulated by the Inventory Manager, O_I, and the Repair Survival Rate, RSR. Therefore it seems reasonable that R_{IR}(t-PCLT,t-RTAT) is independent of α.

Combining equations (30) and (32) with equation (20), together with the independence assertion, leads to

$$\begin{aligned} Cov[R_{IR}, D'] &= Cov[R_{IR}, Q_R O_I] + Cov[R_{IR}, \alpha] \\ &= Q_R \; Cov[R_{IR}, O_I] \\ &\approx D \; CRR \; RSR(PCLT - RTAT)\,. \end{aligned} \tag{33}$$

Therefore, under conditions where the covariance between B_R and R_{IR} is insignificant, the substitution of equations (19) and (33) into equation (29) leads to

$$\begin{aligned} Var[R(t)] \approx \; & \frac{(Q_R-1)(Q_R+1)}{12} - D \; CRR \; RSR(PCLT-RTAT) \\ & + D \; CRR(PCLT-RTAT)\,; \end{aligned} \tag{34}$$

and the variance of net inventory equation (27) becomes

$$\begin{aligned} Var[N(t)] \approx \; & \frac{(Q_R-1)(Q_R+1)}{12} + \frac{(Q_P-1)(Q_P+1)}{12} \\ & + D \; CRR \; RSR \; RTAT + D(1-CRR \; RSR) \, PCLT. \end{aligned} \tag{35}$$

Once again from elementary Queuing Theory results, the variance for the boundary case where Q_R and Q_P are both set equal to one and REP is set equal to zero (being the operation of two independent infinite server queues with Poisson arrivals and fixed service times) is provided by equation (35).

4. Probability Distribution for Net Inventory

The task now is to identify a probability distribution which characterizes the steady state levels of net inventory in the case of instantaneous repair assessment. One way to identify such a distribution is to match moments of the approximating distribution with those determined through stochastic analysis of the conceptual model. For lower order moments, a necessary condition must be that the mean and variance of the chosen distribution are reasonably well approximated by equations (23) and (35), respectively.

After conducting a number of simulation runs for a specific range of conceptual model inputs, Maher [Ref 2:pp. 65-67] found strong evidence to suggest that the Normal distribution was a good approximating distribution for net inventory. Thus, one approach is to model N(t) as a Normal random variable with mean and variance given by equations (23) and (35), respectively. A Normal approximation is likely to allow algebraic simplification in the subsequent development of a readiness based inventory model for repairables. However, it also carries with it the condition that the probability that net inventory is larger than SW is strictly positive; a condition that violates the conceptual model.

A second approach to approximating the distribution of net inventory is to let net inventory be described by

$$N(t) = SW - [B_R(t) + B_P(t) + Y(t)], \qquad \textbf{(36)}$$

where $B_R(t)$ and $B_P(t)$ are as described previously, and Y(t) is a Poisson random variable with mean μ defined as

$$\mu = D\ CRR\ RSR\ RTAT + D(1-CRR\ RSR)\ PCLT. \qquad \textbf{(37)}$$

Then the approximating distribution for N(t), based on equation (36), would be the convolution of three independent random variables $B_R(t)$, $B_P(t)$ and Y(t), and would have its mean and variance described by equations (23) and (35), respectively. In this approach, since $B_R(t)$, $B_P(t)$ and Y(t) are all positive random variables, the probability that N(t) is greater than SW is strictly zero.

The probability density function for this convolution is

$$P[N(t) = SW-z] = \begin{cases} \dfrac{1}{Q_P Q_R} \displaystyle\sum_{j=0}^{z} \dfrac{(j+1)\mu^{z-j}e^{-\mu}}{(z-j)!} & \text{for } 0 \le z \le x_1 \\[2ex] \dfrac{1}{Q_P Q_R} \displaystyle\sum_{j=0}^{x_1} \dfrac{(j+1)\mu^{z-j}e^{-\mu}}{(z-j)!} + \dfrac{\min(Q_P, Q_R)}{Q_P Q_R} \displaystyle\sum_{j=x_1+1}^{z} \dfrac{\mu^{z-j}e^{-\mu}}{(z-j)!} & \text{for } x_1 < z \le x_2 \\[2ex] \dfrac{1}{Q_P Q_R} \displaystyle\sum_{j=0}^{x_1} \dfrac{(j+1)\mu^{z-j}e^{-\mu}}{(z-j)!} + \dfrac{\min(Q_P, Q_R)}{Q_P Q_R} \displaystyle\sum_{j=x_1+1}^{x_2} \dfrac{\mu^{z-j}e^{-\mu}}{(z-j)!} + \dfrac{1}{Q_P Q_R} \displaystyle\sum_{j=x_2+1}^{\min(x_{Max}, z)} \dfrac{(x_{Max}+1-j)\mu^{z-j}e^{-\mu}}{(z-j)!} & \text{for } x_2 < z; \end{cases} \tag{38}$$

where x_{Max}, x_1, and x_2 were defined by equation (5).

IV. NON-INSTANTANEOUS REPAIR ASSESSMENT

This chapter considers the case where the time element REP in the conceptual model assumes some positive value. Under this condition, as a repair batch of size Q_R reaches a repair agency, each item in the batch is sequentially assessed before proceeding into repair or being declared an attrition. Thus, carcasses enter repair agencies in batches but they are assessed individually and, if repaired, returned to wholesale stock as individual items.

Generally speaking, repairable item inventory models discussed in the literature assume that this repair assessment happens instantaneously [Ref. 6 and 8] (as presented in the previous chapter), and/or that repair orders sent to repair agencies are assessed as a single batch and returned as a batch after repair [Ref 5,6,7 and 8]. Such conditions combined with manageable batch arrival distributions obviously have resulted in tractable results. However, the method of repair assessment and return of repaired units outlined in the conceptual model and described above, together with the batching of repair orders initially incorporated in Chapter III, distinguish the Navy model from these other repairable models discussed in the literature.

The nature of the conceptual model (with the condition of non-instantaneous repair assessment) has not made it possible in this study to develop a strong analytical argument leading to the probability distribution of net inventory. However, building on the arguments presented for the instantaneous repair assessment case, an approximation for the mean of net inventory can be devised and a simplistic approach applied to approximate the distribution.

The first section of this chapter briefly considers the steady state distribution of inventory position. The second section considers the net inventory level for the case of non-instantaneous repair assessment, leading to a suggested probability distribution for net inventory.

A. INVENTORY POSITION

Maher [Ref 2:p.59] found that for the range of system parameters he considered the variation of REP had little effect on the simulated inventory position distribution. It remained essentially that stated by equation (4). Thus, a starting point in this analysis is to choose to model inventory position in the same way as was done in the case of instantaneous repair assessment. Specifically, the steady state distribution of inventory position in the case of non-instantaneous assessment is assumed to be approximated by equations (4) and (5). The results of this thesis to be presented later support Maher's results and this general approach for modeling inventory position.

B. NET INVENTORY LEVELS

To consider net inventory levels let $N(t)$, $D(t_1,t_2)$, $D'(t_1,t_2)$, $D''(t_1,t_2)$, and $R_{IR}(t_1,t_2)$ be as defined previously, and let

$A_A(t)$ = number of attritions previously inducted into repair agencies and still awaiting repair assessment at time t; and

$A_R(t)$ = number of repairable carcasses previously inducted into repair agencies and still awaiting repair assessment at time t.

As with the case of instantaneous repair assessment, the general conditions apply that under the assumption that RTAT is strictly less than PCLT in the time period (t-PCLT,t)

- Any quantity in procurement or actual repair at time t-PCLT will have been received by time t.
- Any repairable carcasses commencing actual repair in the period (t-PCLT,t-RTAT) will have been received by time t.

Accounting for the fact that carcasses entering repair agencies now may be delayed by the repair assessment action before being declared repairable or attritions, it follows that

$$N(t) = I(t\text{-}PCLT) - D(t\text{-}PCLT, t) - A_A(t\text{-}PCLT) + R_{IR}(t\text{-}PCLT, t\text{-}RTAT) - A_R(t\text{-}RTAT)\,; \tag{39}$$

which, after substituting equation (1) for I(t-PCLT) becomes

$$N(t) = SW - B_R(t\text{-}PCLT) + R_{IR}(t\text{-}PCLT, t\text{-}RTAT) - D(t\text{-}PCLT, t) - A_A(t\text{-}PCLT) - A_R(t\text{-}RTAT) - B_P(t\text{-}PCLT\,)\,. \tag{40}$$

It should be noted that when REP is set to zero then A_A and A_R are necessarily zero, and equation (40) reduces to equation (9).

1. Repairable Carcasses (A_R) and Attritions (A_A) At Repair Agencies Awaiting Repair Assessment

The robust results of Little [Ref. 12] can be used to study the number of repairable carcasses and attritions at repair agencies awaiting repair assessment. Little showed that in a steady-state queuing process the following formula holds

$$L = \lambda W; \tag{41}$$

where

L = expected number of units in the system;

W = expected time spent by a unit in the system; and

λ = expected rate of arrivals to the system.

The robustness of this result lies in the observation that it is

> ... remarkably free of specific assumptions about arrival and service distributions, independence of inter-arrival times, number of channels, queue disciplines, etc. [Ref. 12:p.387]

It was stated previously that the number of repairable carcasses entering repair agencies by time t $\{R_{IR}(0,t),\ t>0\}$ can be modeled as a Reward Renewal Process or Cumulative Process. In particular, Renewal Theory results for this type of process lead to

$$E[R_{IR}(0,t)] \approx D\ CRR\ RSR\ t, \tag{42}$$

as derived in equation (16) in Chapter III. Thus, choosing to approximate the steady state rate of repairable carcasses entering repair agencies as D*CRR*RSR and defining

T_A = time a randomly selected item spends at a repair agency awaiting repair assessment,

it follows from Little's formula that

$$E[A_R(t)] = D\ CRR\ RSR\ E[T_A]\,. \tag{43}$$

Similar arguments can be made for the number of attritions entering repair agencies by time t such that

$$E[A_A(t)] = D\ CRR\ (1\text{-}RSR)\ E[T_A]\,. \tag{44}$$

2. Time Awaiting Repair Assessment (T_A)

To determine the time any carcass or attrition spends at a repair agency awaiting repair assessment (T_A), define the random variables

I = the sequence number of an item in a repair batch of size Q_R (1, 2... Q_R); and

R_A = the number of items in a particular repair batch of size Q_R assessed as repairable prior to an item undergoing repair assessment (0, 1... Q_R-1).

The conceptual model states that as a repair batch of size Q_R enters a repair agency the first item in the batch is assessed immediately. The outcome of this assessment directly influences the amount of time the remainder of that batch spends waiting for repair assessment to commence. In contrast, the assessment of the last item in the batch does not influence the time any item spends awaiting repair assessment. More generally, the time any randomly selected item spends waiting for repair assessment to commence will be an integer sum of time packages REP and will depend on the number of repairable items in the batch of size Q_R that are assessed before it. This can be represented as the product

$$T_A = R_A(REP)\,. \tag{45}$$

Then, conditioning on the sequence number of an item, the conditional probability becomes

$$P(R_A=x/I=i) = \binom{i-1}{x}(RSR)^x(1-RSR)^{i-1-x} \quad for\ 0 \le x \le i-1; \qquad \textbf{(46)}$$

where any position in the order is equally likely; i.e.,

$$P(I=i) = \frac{1}{Q_R}. \qquad \textbf{(47)}$$

Then the expected time any item spends at a repair agency awaiting repair assessment will be determined as follows:

$$E[T_A] = REP\ E[R_A]; \qquad \textbf{(48)}$$

$$E[R_A] = E[\ E[R_A/I]\ ]; \qquad \textbf{(49)}$$

$$E[R_A/I=i]=(i-1)(RSR); \qquad \textbf{(50)}$$

$$E[R_A]=\sum_{i=1}^{Q_R}(i-1)RSR\ P(I=i) = RSR\ \frac{Q_R-1}{2}; \qquad \textbf{(51)}$$

$$E[T_A]=\frac{Q_R-1}{2}\ RSR\ REP. \qquad \textbf{(52)}$$

3. Expected Net Inventory

Relaxing the time interval notation and taking the expectation of N(t) given by equation (40) leads to

$$\begin{aligned} E[N(t)] &= SW - E[B_R] + E[R_{IR}] - E[D] \\ &\quad - E[A_A] - E[A_R] - E[B_P]. \end{aligned} \qquad \textbf{(53)}$$

Substituting the results obtained from equations (6), (16), (43) and (44) leads to

$$E[N(t)] \approx SW - \frac{Q_R-1}{2} - \frac{Q_P-1}{2} - D(1-CRR\ RSR)\,PCLT - D\ CRR(RSR\ RTAT + E[T_A])\,. \tag{54}$$

Finally, adding the result given by equation (52) gives

$$E[N(t)] \approx SW - \frac{Q_R-1}{2} - \frac{Q_P-1}{2} - D(1-CRR\ RSR)\,PCLT - D\ CRR\ RSR(RTAT + \frac{(Q_R-1)\ REP}{2})\,. \tag{55}$$

It should be noted that when REP is set to zero equation (55) reduces to equation (23) in Chapter III.

4. Variance of Net Inventory

The structure of N(t) given by equation (35) allows for the direct application of the arguments used in the previous chapter to the determination of variance of net inventory in the case of non-instantaneous repair assessment. However, point estimates for the variances of A_A and A_R , and the form of any dependencies induced by these random variables, are not obvious.

While formulae for higher order moments similar to Little's result do exist in the case of Exponential inter-arrival patterns [Ref. 13], they are ineffective in this case as in the conceptual model, Erlang inter-arrival patterns of repair batches at an infinite number of repair agencies are experienced. Then, on arrival at a repair agency, each batch enters a single server queue for repair assessment. The inability to accurately model this process leads to consideration of the following naive analysis.

In the boundary cases where the Chapter III assumption concerning the insignificance of $Cov[B_R, R_{IR}]$ is applied, Q_R is set equal to one, and/or REP is set equal to zero, any expression for the variance of net inventory should equate to equation (35). Similarly, as REP approaches zero the point estimate results should approach those for the instantaneous repair assessment case. Furthermore, for REP sufficiently small, it might be expected that the lower order moments for the deviation of net inventory below SW approach those of a random variable formed from the convolution of $B_R(t)$, $B_P(t)$ and a Poisson random variable with mean

$$\mu = D\,(1-CRR\;RSR)\;PCLT + D\;CRR\;RSR\left(RTAT + \frac{(Q_R-1)\;REP}{2}\right). \tag{56}$$

With this naive analysis the variance of net inventory would be estimated by

$$Var[N(t)] \approx \frac{(Q_R-1)(Q_R+1)}{12} + \frac{(Q_P-1)(Q_P+1)}{12} + D\;CRR\;RSR\;\left(RTAT + \frac{(Q_R-1)\;REP}{2}\right) + D(1-CRR\;RSR)\,PCLT. \tag{57}$$

5. Probability Distribution for Net Inventory

Once again using the approach of matching lower order moments, and applying the results of the naive analysis, an approximation can be made for the steady state distribution of net inventory.

Maher found that the Normal distribution served as a good approximation for net inventory for both instantaneous and non-instantaneous repair assessment. Thus, one approach is to model N(t) as a Normal random variable with mean and variance given by equations (55) and (57), respectively.

A second approach, identical to that followed in Chapter III, is to let net inventory be described by

$$N(t) = SW - [B_R(t) + B_P(t) + Y(t)]; \qquad (58)$$

where $B_R(t)$ and $B_P(t)$ are as describe previously, and Y(t) is a Poisson random variable with mean given by equation (56). Then the approximating distribution for N(t) resulting from equation (58) and the convolution of three independent random variables $B_R(t)$, $B_P(t)$ and Y(t) will have mean and variance described by equations (55) and (57), respectively. The probability distribution function for this convolution will be that given by equation (38) with μ now given by equation (56).

V. RESULTS

The purpose of this chapter is to discuss the performance of the approximations developed for inventory position and net inventory in Chapters III and IV. The first section of the chapter outlines the methods used to test the distributional approximations developed previously. The second section of the chapter discusses results obtained in the instantaneous repair assessment case. Finally, the third section discusses the results for the case of non-instantaneous repair assessment.

A. TESTING METHOD

The objective of this thesis is to develop probability distributions which describe the steady state behavior of the Navy's wholesale repairable item inventory. These distributions can then be used in an inventory management model for repairable items. Typically such models seek to optimize a selected measure of effectiveness such as total variable cost of maintaining inventory, supply response time, material availability, etc. Thus, the most appropriate form of testing would be to investigate how well the suggested probability distributions work in such a model. In the absence of an inventory management model a more stringent test is to investigate how well the approximate analytical probability distributions for inventory position and net inventory compare with empirical distributions generated through simulation. If the results are satisfactory then the distributions can be used in the development of an inventory management model. This later approach was adopted in this thesis.

To validate the probability distributions for inventory position and net inventory which were described in Chapters III and IV, a number of simulation runs were conducted using the MODSIM II high-level programming language. MODSIM II is a general-purpose, modular, object-oriented programming language which has built-in capability to perform discrete event simulation [Ref 14: pp.1-2]. The MODSIM II simulation implements the conceptual model described in Chapter II as was that previously studied by Maher [Ref. 2] using SIGMA (SImulation Graphical Modeling and Analysis system). The simulation output published by Maher therefore provided an initial verification tool for the MODSIM II program used in this thesis.

For the range of system inputs considered, and program run times experienced, an appropriate simulation length was considered to be 1000 quarters. Based on analysis of simulation time series an initial transient time of 50 quarters was generally applied.

The simulation runs were used to study the performance of point estimates and distribution approximations using a combination of graphical techniques and goodness of fit tests. As with Maher's thesis, for the Chi-square goodness of fit test the "observed frequencies" resulting from the simulation model were determined by comparing the total length of time that each value of the inventory position and net inventory occurred in steady state with the total run time less initial transient time (i.e., the time-weighted distributions were determined). As Maher noted [Ref. 2:p. 58], this use of the Chi-square goodness of fit test stretches the theory and is not exact. However, no other goodness of fit test exists for this type of frequency determination.

B. INSTANTANEOUS REPAIR ASSESSMENT

1. Point Estimates

As an initial illustration of the effectiveness of the approximations developed for the particular case of instantaneous repair assessment a number of system input settings were considered. These settings fall within the range of system input values discussed in Chapter II, and are shown in Table 1.

Point estimates for the mean and variance of inventory position and net inventory for these five system input settings are shown in Tables 2 and 3, respectively. In all cases it can be seen that the approximations are very close to those obtained from simulation runs. In particular, over a large number of additional runs the simulated results gave a mean and variance of inventory position and a mean of net inventory which were always very close to the approximate formulae. Simulation output for the variance of net inventory tended to be less close to the approximate formulae.

Case No.	D	CRR	RSR	Q_R	Q_P	RTAT	PCLT
1	30	0.7	0.7	20	40	1.0	5.0
2	5	0.8	0.75	6	10	1.2	5.5
3	2	0.85	0.77	3	5	1.3	6.0
4	1	0.9	0.8	2	4	1.1	7.0
5	0.5	0.95	0.85	1	2	1.6	8.0
All cases use SW = 72, REP = 0.0							

Table 1. System Input Settings.

2. Steady State Distribution of Inventory Position

A graphic comparison of the simulated and approximating probability distributions for inventory position resulting from the application of input settings in Table 1 is contained in Appendix B. These initial results support Maher's findings [Ref 2:p.78] that the approximation developed provides a very good fit.

Case No.	Simulation		Approximation	
	Mean	Variance	Mean	Variance
1	42.473	163.528	43.0	166.5
2	64.841	10.984	65.0	11.16
3	68.936	2.772	69.0	2.66
4	69.927	1.578	70.0	1.50
5	71.452	0.248	71.5	0.25

Table 2. Comparison of Inventory Position Point Estimates.

Case No.	Simulation		Approximation	
	Mean	Variance	Mean	Variance
1	-47.005	240.235	-48.2	257.866
2	50.483	23.743	50.4	25.7
3	63.208	8.261	63.152	8.514
4	67.440	3.847	67.248	4.252
5	69.999	1.633	70.084	1.666

Table 3. Comparison of Net Inventory Point Estimates.

Appendix C contains results of Chi-Square goodness of fit test for 78 simulation runs applying a range of system inputs. In all cases the observed test statistic, χ^2, is less than the critical value, $\chi^2_{.95,\upsilon}$. Thus, at the 5% significance level the conclusion is to fail to reject the null hypothesis that the distribution of inventory position is as described by equations (4) and (5).

3. Steady State Distribution of Net Inventory

Graphical comparisons of the simulated distributions for net inventory and Normal approximations resulting from the application of input settings in Table 1 to the simulation model are shown in Appendix D, and the convolution approximations are shown in Appendix E. In these initial cases it is difficult to discern any significant difference between the graphics for the Normal and convolution approximations. Both seem to provide a very good representation of the simulated output. However, it should be noted that the graphics for the Normal approximation do not include theoretical probabilities for net inventory levels greater than SW (72 in all cases) because net inventory levels will never exceed SW in the conceptual model.

Appendices F and G contain the Chi-Square goodness of fit test results for 78 simulation runs using the Normal and convolution approximations, respectively. Once again at the 5% level of significance the general conclusion is to fail to reject the null hypothesis that the distribution of net inventory is described by the Normal approximation with mean and variance given by (23) and (35), respectively, or by the convolution given by equation (38). It is only in the cases where low variability of net inventory is experienced in steady state (reflected in the degrees of freedom for the Chi-square test which in all cases are equivalent to the number of inventory levels experienced in steady state less one) that the Normal approximation appears to consistently fail. This failure is likely to be the result of the Normal approximation applying significant positive probability to net inventory levels greater than SW. The convolution approximation is accepted in all cases.

The variance approximation derived in Chapter III and applied in cases 1 through 5 above has assumed that the covariance between B_R(t-PCLT) and R_{IR}(t-PCLT,t-RTAT) is insignificant. Given the relationship developed in equation (29), and the intuitive reasoning developed in Chapter III supporting positive correlation between these two random variables, it follows that the variance approximation should be greater than the variance reflected in simulation results. Indeed, this phenomena was consistently observed over the large number of simulation runs performed during this study. Despite this result, the convolution approximation for the steady state distribution of net inventory seems to work very well and is supported by the Chi-square goodness of fit test results.

C. NON-INSTANTANEOUS REPAIR ASSESSMENT

1. Steady State Distribution of Inventory Position

Appendix H contains the Chi-square goodness of fit results for the approximating distribution of net inventory following the application of a range of system input settings to the simulation model. In these cases REP varies from 0.0 to 0.5. Once again the observed test statistic χ^2 is continually, and significantly, less than the critical value, $\chi^2_{.95,\upsilon}$. Thus, at the 5% significance level the conclusion is to fail to reject the null hypothesis that the distribution of inventory position is as described by equations (4) and (5).

These results together with those obtained in the instantaneous repair assessment case add further weight to Maher's findings; that the convolution of two discrete Uniform random variables provides a robust approximation for the steady state distribution of inventory position.

2. Net Inventory Point Estimates

To assess the approximations developed for net inventory the theoretical point estimates were first compared to simulation output using the input settings in Table 4. Graphical comparisons of the theoretical and simulated mean and variance of net inventory are at Appendices I and J, respectively. Each input setting was replicated four times in the simulation model.

Case No.	D	CRR	RSR	REP	Q_R	Q_P	RTAT	PCLT
6	20	0.7	0.7	0.0-0.4	20	30	1.0	4.0
7	9	0.65	0.7	0.0-1.0	7	8	0.9	4.0
8	5	0.7	0.75	0.0-1.2	4	10	1.2	5.0
9	2	0.8	0.7	0.0-1.5	3	5	1.3	6.0
10	1	0.85	0.8	0.0-1.6	2	4	1.6	6.5
All cases use SW = 72								

Table 4. System Input Settings.

As with the case of instantaneous repair assessment, the theoretical estimates for the expected net inventory are very close to those gained from the simulation. However, in the case of the variance the theoretical point estimate consistently overstates the result obtained through simulation. This result is not surprising given the simplistic basis for the variance estimate. The simulation results also show that variance as a distributional parameter is not particularly well behaved. The nature of this behavior presents difficulties in trying to determine a general estimate for the difference between the simulated and estimated variances using regression techniques.

3. Steady State Distribution of Net Inventory

Despite the shortcomings displayed in the variance estimates for net inventory, the Normal and convolution distributional approximations were tested against the simulation model. Appendices K and L contain the Chi-Square goodness of fit test results for 90 simulation runs applying a range of system input settings and using the Normal and convolution approximations, respectively. Once again at the 5% level of significance the general conclusion is to fail to reject the null hypothesis that the distribution of net inventory is described by the convolution approximation given by equation (38). The Normal approximation, however, continues to consistently fail the hypothesis test where low variability of net inventory is experienced. Note that, as expected from the conceptual model, in the case where Q_R equals one the value of REP has no influence on the distributional parameters.

The simulation runs conducted in Appendices K and L with REP equal to 0.5 (its largest value in Appendices K and L) were used to provide a graphical comparison of the net inventory probabilities obtained through simulation and the convolution approximation as given by equation (38). These plots are contained in Appendix M, with case input settings summarized in Table 5. The plots show that the convolution resembles the simulated result quite closely, with the absolute differences between the simulated and approximated probabilities being very small in all cases. While the plots for Cases 18 and 19 appear to show that the two distributions begin to differ, it should be noted that in these cases the relative shape of the distribution function is extremely flat and the absolute differences in probabilities remain very small. The Chi-square test for these two cases (see last two results in Appendix L) leads to the conclusion, at the 5% significance level, that the convolution is a reasonable representation of the steady state distribution of net inventory.

Case No.	D	CRR	RSR	Q_R	Q_P	RTAT	PCLT
11	1	0.56	0.86	1	1	0.9	8.21
12	2	0.7	0.62	3	2	0.7	4.55
13	3	0.72	0.94	4	7	0.98	4.05
14	4	0.64	0.42	8	8	1.17	5.04
15	6	0.73	0.94	8	14	1.58	6.76
16	8	0.53	0.43	8	20	2.17	6.12
17	12	0.43	0.81	9	14	0.94	3.64
18	16	0.43	0.8	24	20	1.14	7.59
19	20	0.86	0.91	18	22	1.51	5.43
All cases use SW = 72, REP = 0.5							

Table 5. System Input Settings.

VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A. SUMMARY

In the past the Navy has managed substantial inventories of both consumable and repairable secondary items. However, changes implemented in the Department of Defense wholesale supply management as a consequence of Defense Management Review Decision 926 have seen a large migration of the consumable items to DLA and leaves the Navy and the other services to focus their inventory management efforts on repairable items. In addition, readiness-based sparing is becoming increasingly important as a result of Congressional pressures. To assist in this adjustment of focus the Navy requires tools which help in the determination of optimal strategies for the maintenance and control of a repairable item inventory.

The Navy's current repairable item inventory control model has been found to give unsatisfactory results when attempts are made to use it for readiness-based sparing. As a consequence, there is a requirement for a fundamental analysis of the Navy's repairable item inventory management process, rather than a simple patch on the existing model. In light of this, work has been undertaken at the Naval Postgraduate School to develop a new mathematical model for wholesale level repairable item inventory control.

The first step in the development process is to determine probability distributions which describe the steady state behavior of a repairable item inventory. The complex mix of stochastic processes which occur in the life of a repairable item has discouraged previous analytical approaches. This has led to the application of simulation techniques to determine approximately correct distributions.

This thesis has shown that it is possible to use the simulation results to suggest and to test an analytical approach for the development of approximate probability distributions for inventory position and net inventory. The repairable item inventory process was analyzed for the two separate cases of instantaneous and non-instantaneous repair assessment. Using established stochastic modeling results and the previously derived approximation for inventory position, approximations for the mean and variance of net inventory were developed. Two approximating distributions for net inventory were proposed and their performance along with that of the inventory position approximation were tested against results obtained through simulation.

B. CONCLUSIONS

1. Inventory Position

The results of this thesis support Maher's findings [Ref.2] that the convolution of two discrete Uniform random variables provides a robust approximation for the steady state distribution of inventory position. This result holds for both the instantaneous and the non-instantaneous repair assessment cases over a wide range of system inputs.

2. Net Inventory

The approximations developed in this thesis for the mean of net inventory closely matched simulation output in both the instantaneous and non-instantaneous repair assessment cases. The variance estimates consistently overestimated the simulation results, however. A major reason for this appears to be the assuming away of some random variable dependencies (i.e., covariances were assumed to be zero). Despite these results, the Normal and convolution approximations proposed for the steady state distribution of net inventory appeared to closely resemble the simulation output. On the basis of the graphical analyses and goodness of fit testing the convolution approximation seems a robust approximation for the net inventory distribution over a wide range of system inputs.

C. RECOMMENDATIONS

This thesis represents a significant step in the development of a repairable item inventory control model for the Navy. The next logical step is for the approximations developed in this thesis to be compared against inventory characteristics observed for actual repairable items being managed by the Navy. In particular, a formula for the expected number of time-weighted backorders at any instant of time needs to be developed and the values of average days delay it computes compared to those being observed by the Navy's Inventory Control Point repairable item inventory managers. Subject to a favorable outcome to this comparison a readiness-based sparing repairable item inventory model using the approximations developed in this thesis should be readily forthcoming.

APPENDIX A. SUMMARY OF SYSTEM INPUTS AND RANDOM VARIABLES

System Inputs

SW	Maximum inventory level
D	Quarterly demand rate
CRR	(Carcass Return Rate) Probability that a demand will be accompanied by a carcass
RSR	(Repair Survival Rate) Probability that a carcass (failed item) is repairable
Q_R	Number of items in a repair batch/order
Q_P	Number of items in a procurement batch/order
REP	The time interval that elapses in the repair assessment process following the assessment of a repairable item until the assessment of the next carcass in the order
RTAT	(Repair Turn Around Time) The time taken by a repair agency to repair an individual item including repair assessment time
PCLT	(Procurement Lead Time) The time taken by a supplier to fill a procurement order

Random Variables

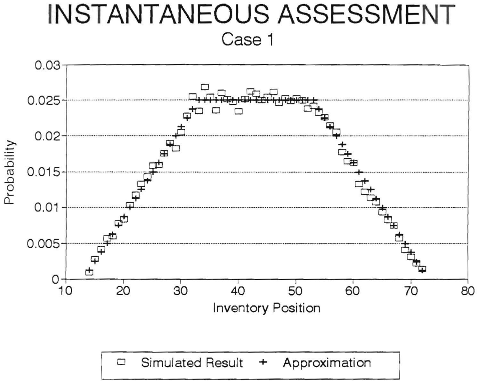
$I(t)$	=	the inventory position at time t
$B_R(t)$	=	the number of carcasses in the repair batching process at time t
$B_P(t)$	=	the number of attritions in the procurement batching process at time t
$N(t)$	=	net inventory level at time t
$D(t_1,t_2)$	=	number of demands in the period (t_1,t_2)

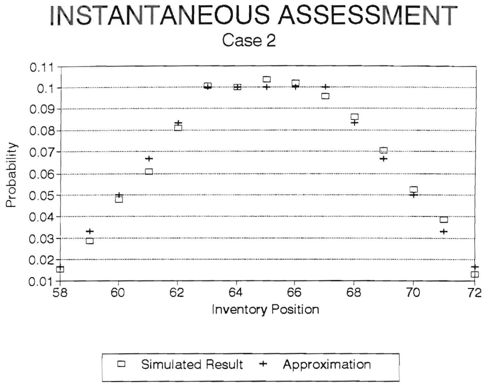
$D'(t_1,t_2)$	=	number of demands in the period (t_1,t_2) accompanied by a carcass
$D''(t_1,t_2)$	=	number of demands in the period (t_1,t_2) not accompanied by a carcass
$R_{IR}(t_1,t_2)$	=	number of carcasses inducted into repair agencies in the period (t_1,t_2) that will be assessed as repairable
T_{RB}	=	the time between the despatching of successive repair batches of size Q_R to a repair agency
$O_I(t)$	=	the number of repair batches of size Q_R inducted into repair agencies in the time interval $[0,t)$
$O_I(t_1,t_2)$	=	the number of repair batches of size Q_R inducted into repair agencies in the time period (t_1,t_2)

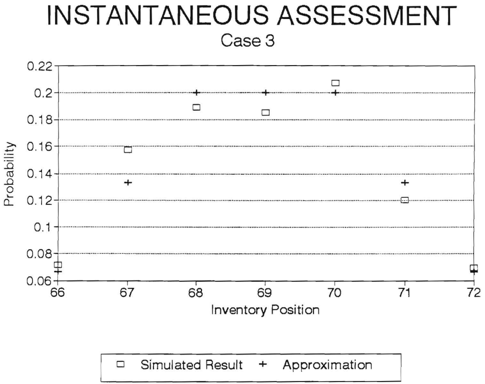
Random Variables - Non-Instantaneous Repair Assessment Only

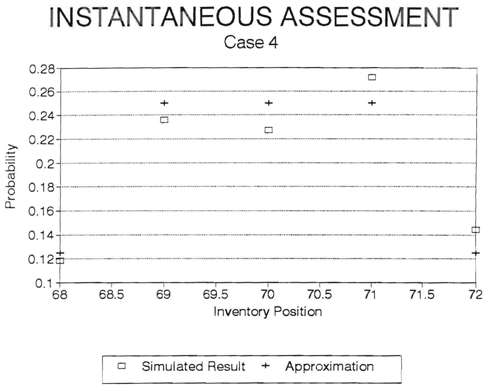
$A_A(t)$	=	number of attritions previously inducted into repair agencies and still awaiting repair assessment at time t
$A_R(t)$	=	number of repairable carcasses previously inducted into repair agencies and still awaiting repair assessment at time t
T_A	=	time an item spends at a repair agency awaiting repair assessment
I	=	the sequence number of an item in a repair batch of size Q_R (1, 2 ... Q_R)
R_A	=	the number of items in a particular repair batch of size Q_R assessed as repairable prior to an item undergoing repair assessment (0, 1 ... Q_R-1)

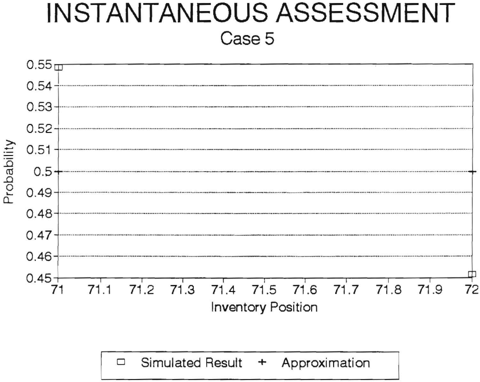
APPENDIX B. COMPARISON OF PROBABILITY DISTRIBUTIONS FOR INVENTORY POSITION WITH INSTANTANEOUS REPAIR ASSESSMENT




INSTANTANEOUS ASSESSMENT
Case 2
Probability
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
58
60
62
64
66
68
70
72
Inventory Position
Simulated Result
Approximation


INSTANTANEOUS ASSESSMENT
Case 3
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
Probability
66
67
68
69
70
71
72
Inventory Position
Simulated Result
Approximation


INSTANTANEOUS ASSESSMENT
Case 4
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
Probability
68
68.5
69
69.5
70
70.5
71
71.5
72
Inventory Position
Simulated Result
Approximation


INSTANTANEOUS ASSESSMENT
Case 5
0.55
0.54
0.53
0.52
0.51
0.5
0.49
0.48
0.47
0.46
0.45
Probability
71
71.1
71.2
71.3
71.4
71.5
71.6
71.7
71.8
71.9
72
Inventory Position
Simulated Result
Approximation

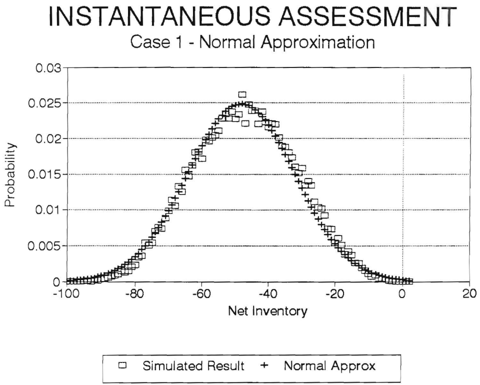
APPENDIX C. GOODNESS OF FIT TEST RESULTS FOR INVENTORY POSITION WITH INSTANTANEOUS REPAIR ASSESSMENT

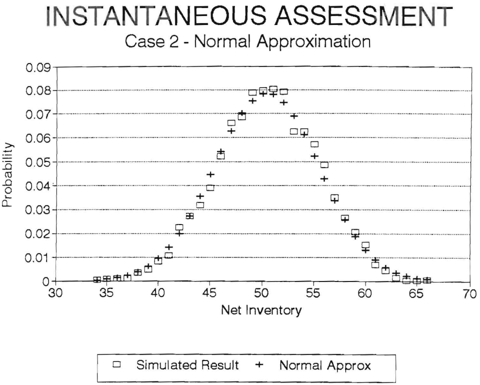
D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
1	0.56	0.86	0.9	8.21	1	1	0	0	0
1.2	0.93	0.97	2.03	3.8	2	2	4.376772	5.99147	2
1.4	0.57	0.88	1.23	7.18	3	3	4.210447	9.48773	4
1.6	0.4	0.86	1.21	5.7	3	4	0.875907	1.0705	5
1.8	0.46	0.91	0.98	4.07	4	7	7.337342	16.919	9
2	0.7	0.62	0.7	4.55	3	2	0.82123	7.81473	3
2.2	0.55	1	2.3	8.04	4	4	2.489711	12.5916	6
2.4	0.55	0.94	0.99	7.43	3	6	4.671086	14.0671	7
2.6	0.67	0.65	0.6	3.99	2	6	3.48185	12.5916	6
2.8	0.81	0.73	2.08	7.83	4	8	8.605134	18.307	10
3	0.72	0.94	0.98	4.05	4	7	6.688924	16.919	9
3.2	0.77	0.68	1.79	7.61	4	10	3.268578	21.0261	12
3.4	0.54	0.48	2.26	5.04	5	10	3.55471	22.3621	13
3.6	0.74	0.68	1.51	6.61	7	8	4.930263	22.3621	13
3.8	0.77	0.93	1.21	4.28	6	9	3.899164	22.3621	13
4	0.64	0.42	1.17	5.04	8	8	5.562638	23.6848	14
4.2	0.83	0.44	0.54	6.11	4	6	3.731272	15.5073	8
4.4	0.65	0.94	2.27	4.57	7	7	5.118714	21.0261	12
4.6	0.91	0.9	1.11	7.5	6	15	7.146105	30.1435	19
4.8	0.9	0.98	0.38	7.94	5	20	9.674624	35.1725	23
5	0.67	0.51	0.31	5.25	5	12	4.146568	24.9958	15
5.2	0.93	0.6	2.06	7.71	7	24	12.79699	42.5569	29
5.4	0.76	0.6	0.51	4.61	8	10	3.918378	26.2962	16
5.6	0.7	0.5	2.24	5.57	6	6	4.317124	18.307	10
5.8	0.82	0.98	0.53	7.09	7	6	2.271654	19.6751	11
6	0.73	0.94	1.58	6.76	8	14	6.151449	31.4104	20

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
6.2	0.41	0.93	1.81	6.08	8	8	3.82707	23.6848	14
6.4	0.79	0.48	1.61	6.09	10	8	1.821077	26.2962	16
6.6	0.97	0.44	1.37	6.01	5	24	6.411615	40.1133	27
6.8	0.88	0.66	0.52	8.26	4	10	1.49633	21.0261	12
7	0.5	0.8	2	4.85	6	18	4.506006	33.9244	22
7.2	0.82	0.77	1.79	5.02	12	22	12.42674	46.1942	32
7.4	0.81	0.89	0.36	7.92	9	12	1.782152	30.1435	19
7.6	0.47	0.58	0.38	8.2	10	8	3.042209	26.2962	16
7.8	0.79	0.81	0.73	8.41	7	10	2.121086	24.9958	15
8	0.53	0.43	2.17	6.12	8	20	4.087285	38.8852	26
8.2	0.8	0.93	2.06	7.9	13	15	3.196558	38.8852	26
8.4	0.96	0.67	0.66	5.96	4	4	1.778286	12.5916	6
8.6	0.4	0.74	1.55	5.74	9	12	3.109613	30.1435	19
8.8	0.54	0.6	0.52	3.51	10	9	1.351535	27.5871	17
9	0.68	0.6	1	6.56	5	10	1.522632	22.3621	13
9.5	0.78	0.84	1.06	7.25	11	20	3.074357	42.5569	29
10	0.73	0.61	1.52	4.9	8	5	0.772842	19.6751	11
10.5	0.81	0.9	1.63	4.16	6	10	2.276205	23.6848	14
11	0.56	0.76	0.5	4.4	12	10	3.276276	31.4104	20
11.5	0.64	0.44	0.9	3.85	13	17	2.632506	41.3372	28
12	0.43	0.81	0.94	3.64	14	9	2.23805	32.6705	21
12.5	0.9	0.69	1.25	7.78	9	12	2.456715	30.1435	19
13	0.75	0.45	1.25	5.43	5	5	1.217982	15.5073	8
13.5	0.56	0.57	0.43	7.2	10	15	2.163939	35.1725	23
14	0.66	0.63	0.36	6.48	14	11	2.144016	35.1725	23
14.5	0.81	0.81	2.09	6.37	11	17	2.556159	38.8852	26
15	0.97	0.56	1.8	5.57	15	10	2.204993	35.1725	23
15.5	0.44	0.58	0.88	5.43	17	20	4.160763	49.8019	35
16	0.78	0.82	0.64	4.44	10	8	1.078626	26.2962	16
16.5	0.51	0.65	1.85	4.39	4	8	0.696505	18.307	10
17	0.75	0.81	1.22	5.42	20	14	2.047428	46.1942	32

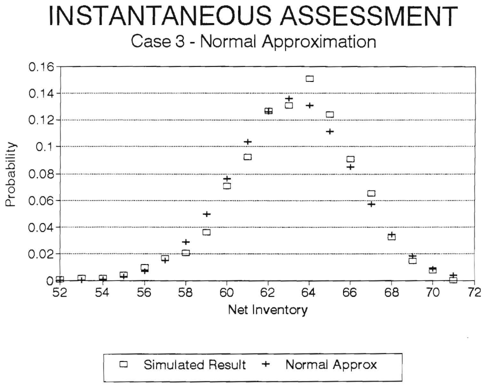
D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
17.5	0.61	0.87	1.19	6.05	17	20	4.58592	49.8019	35
18	0.43	0.8	1.14	7.59	24	20	2.832667	58.1241	42
18.5	0.8	0.7	1.41	4.62	9	8	0.866691	24.9958	15
19	0.94	0.45	1.24	4.54	21	30	4.576058	66.3386	49
19.5	0.8	0.96	1.12	6.54	18	14	2.724438	43.7729	30
20	0.97	0.41	0.82	6.75	14	18	1.900257	43.7729	30
21	0.62	0.63	1.88	4.95	21	24	1.962103	59.3034	43
22	0.62	0.63	0.56	5.42	17	20	3.118684	49.8019	35
23	0.8	0.58	1.32	4.41	30	20	2.285073	65.1706	48
24	0.62	0.9	0.8	3.79	17	30	2.504164	61.6563	45
25	0.41	0.62	0.81	5.41	25	18	1.767047	56.9424	41
26	0.86	0.91	1.51	5.43	18	22	2.079228	53.3837	38
27	0.59	0.87	1.33	4.01	32	24	3.275526	72.1534	54
28	0.43	0.72	1.46	4.66	20	28	2.503216	62.8295	46
29	0.52	0.4	1.64	4.68	15	15	2.008606	41.3372	28
30	0.95	0.83	2.23	4.84	40	10	2.548498	65.1706	48
32	0.79	0.85	0.3	3.42	25	31	1.919398	72.1534	54
34	0.9	0.49	1.7	4.14	42	14	2.812295	72.1534	54
36	0.89	0.96	0.97	3.94	28	40	1.660321	85.965	66
38	0.52	1	1.8	4.2	38	20	2.296568	74.4686	56
40	0.43	0.44	1.43	2.42	34	40	2.627907	92.8085	72

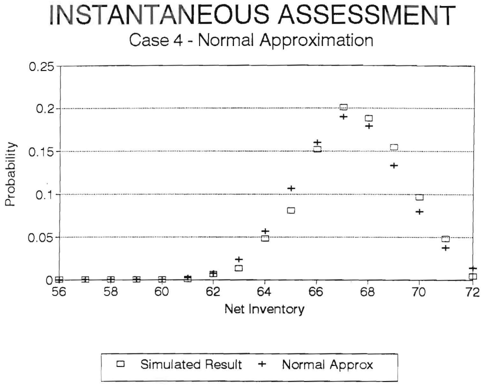
APPENDIX D. COMPARISON OF PROBABILITY DISTRIBUTIONS FOR NET INVENTORY WITH INSTANTANEOUS REPAIR ASSESSMENT - NORMAL APPROXIMATION



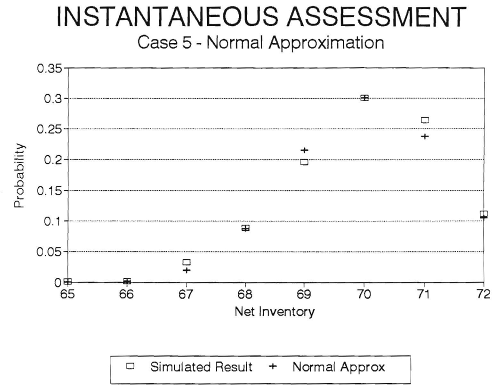


INSTANTANEOUS ASSESSMENT
Case 2 - Normal Approximation
Probability
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
30
35
40
45
50
55
60
65
70
Net Inventory
Simulated Result
Normal Approx

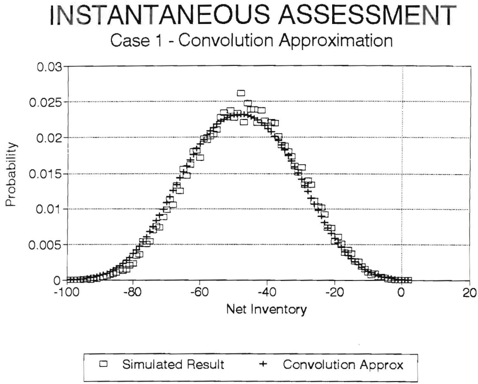

INSTANTANEOUS ASSESSMENT
Case 3 - Normal Approximation
Probability
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
52
54
56
58
60
62
64
66
68
70
72
Net Inventory
Simulated Result
Normal Approx



INSTANTANEOUS ASSESSMENT
Case 4 - Normal Approximation
0.25
0.2
0.15
0.1
0.05
0
Probability
56
58
60
62
64
66
68
70
72
Net Inventory
Simulated Result
Normal Approx

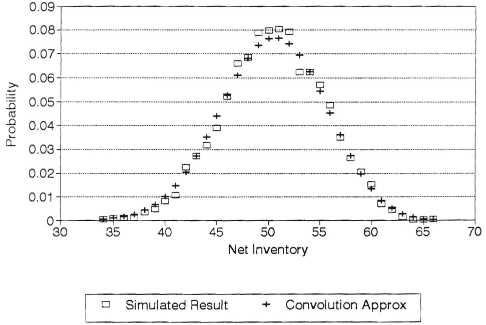

INSTANTANEOUS ASSESSMENT
Case 5 - Normal Approximation
Probability
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
65
66
67
68
69
70
71
72
Net Inventory
Simulated Result
Normal Approx

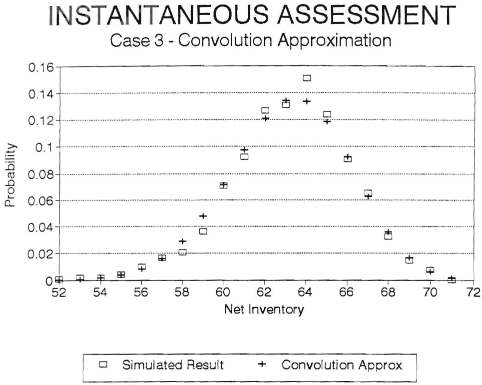
APPENDIX E. COMPARISON OF PROBABILITY DISTRIBUTIONS FOR NET INVENTORY WITH INSTANTANEOUS REPAIR ASSESSMENT - CONVOLUTION APPROXIMATION

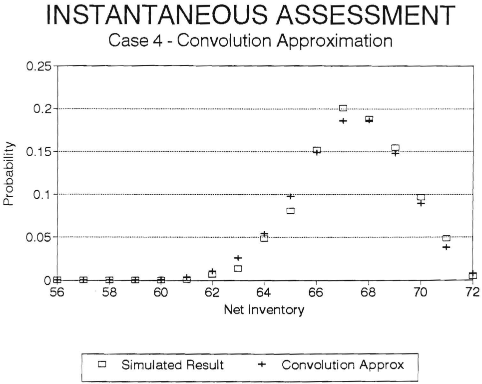


INSTANTANEOUS ASSESSMENT

Case 2 - Convolution Approximation

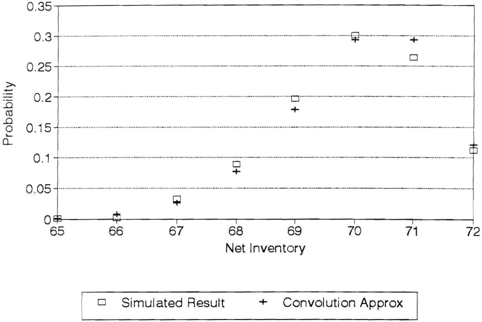



INSTANTANEOUS ASSESSMENT
Case 3 - Convolution Approximation
Probability
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
52
54
56
58
60
62
64
66
68
70
72
Net Inventory
Simulated Result
Convolution Approx


INSTANTANEOUS ASSESSMENT
Case 4 - Convolution Approximation
Probability
0.25
0.2
0.15
0.1
0.05
0
56
58
60
62
64
66
68
70
72
Net Inventory
Simulated Result
Convolution Approx

INSTANTANEOUS ASSESSMENT

Case 5 - Convolution Approximation



APPENDIX F. GOODNESS OF FIT TEST RESULTS FOR NET INVENTORY WITH INSTANTANEOUS REPAIR ASSESSMENT - NORMAL APPROXIMATION

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
1	0.56	0.86	0.9	8.21	1	1	36.437592	22.3621	13
1.2	0.93	0.97	2.03	3.8	2	2	32.577991	16.919	9
1.4	0.57	0.88	1.23	7.18	3	3	34.632537	24.9958	15
1.6	0.4	0.86	1.21	5.7	3	4	29.968999	31.4104	20
1.8	0.46	0.91	0.98	4.07	4	7	20.7532	30.1435	19
2	0.7	0.62	0.7	4.55	3	2	20.661384	28.8693	18
2.2	0.55	1	2.3	8.04	4	4	23.959783	40.1133	27
2.4	0.55	0.94	0.99	7.43	3	6	36.810676	32.6705	21
2.6	0.67	0.65	0.6	3.99	2	6	20.667706	28.8693	18
2.8	0.81	0.73	2.08	7.83	4	8	16.28019	40.1133	27
3	0.72	0.94	0.98	4.05	4	7	11.198242	33.9244	22
3.2	0.77	0.68	1.79	7.61	4	10	28.682596	44.9854	31
3.4	0.54	0.48	2.26	5.04	5	10	18.828359	43.7729	30
3.6	0.74	0.68	1.51	6.61	7	8	46.356123	46.1942	32
3.8	0.77	0.93	1.21	4.28	6	9	12.333993	41.3372	28
4	0.64	0.42	1.17	5.04	8	8	23.19932	43.7729	30
4.2	0.83	0.44	0.54	6.11	4	6	10.994421	42.5569	29
4.4	0.65	0.94	2.27	4.57	7	7	14.800585	49.9854	31
4.6	0.91	0.9	1.11	7.5	6	15	27.655204	44.9854	31
4.8	0.9	0.98	0.38	7.94	5	20	58.173473	47.3998	33
5	0.67	0.51	0.31	5.25	5	12	13.423188	49.8019	35
5.2	0.93	0.6	2.06	7.71	7	24	38.561637	74.4686	56
5.4	0.76	0.6	0.51	4.61	8	10	19.506471	52.1925	37
5.6	0.7	0.5	2.24	5.57	6	6	15.481147	48.6023	34
5.8	0.82	0.98	0.53	7.09	7	6	8.338465	43.7729	30

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
6	0.73	0.94	1.58	6.76	8	14	28.299511	52.1925	37
6.2	0.41	0.93	1.81	6.08	8	8	8.733684	54.5724	39
6.4	0.79	0.48	1.61	6.09	10	8	27.615008	54.5724	39
6.6	0.97	0.44	1.37	6.01	5	24	25.169252	73.3115	55
6.8	0.88	0.66	0.52	8.26	4	10	28.964336	53.3837	38
7	0.5	0.8	2	4.85	6	18	13.705056	68.6693	51
7.2	0.82	0.77	1.79	5.02	12	22	27.683285	67.5048	50
7.4	0.81	0.89	0.36	7.92	9	12	14.626665	54.5724	39
7.6	0.47	0.58	0.38	8.2	10	8	39.001859	62.8295	46
7.8	0.79	0.81	0.73	8.41	7	10	11.751643	55.7585	40
8	0.53	0.43	2.17	6.12	8	20	15.200444	70.9934	53
8.2	0.8	0.93	2.06	7.9	13	15	9.244638	70.9934	53
8.4	0.96	0.67	0.66	5.96	4	4	28.232198	41.3372	28
8.6	0.4	0.74	1.55	5.74	9	12	18.712558	66.3386	49
8.8	0.54	0.6	0.52	3.51	10	9	18.871321	56.9424	41
9	0.68	0.6	1	6.56	5	10	18.70459	60.4808	44
9.5	0.78	0.84	1.06	7.25	11	20	12.601859	73.3115	55
10	0.73	0.61	1.52	4.9	8	5	26.337521	59.3034	43
10.5	0.81	0.9	1.63	4.16	6	10	8.635104	60.4808	44
11	0.56	0.76	0.5	4.4	12	10	25.196976	74.4686	56
11.5	0.64	0.44	0.9	3.85	13	17	18.914689	73.3115	55
12	0.43	0.81	0.94	3.64	14	9	10.496251	67.5048	50
12.5	0.9	0.69	1.25	7.78	9	12	11.393879	73.3115	55
13	0.75	0.45	1.25	5.43	5	5	21.798929	73.3115	55
13.5	0.56	0.57	0.43	7.2	10	15	32.966999	74.4686	56
14	0.66	0.63	0.36	6.48	14	11	48.656652	74.4686	56
14.5	0.81	0.81	2.09	6.37	11	17	10.982619	88.2501	68
15	0.97	0.56	1.8	5.57	15	10	17.363677	73.3115	55
15.5	0.44	0.58	0.88	5.43	17	20	20.652613	92.8085	72
16	0.78	0.82	0.64	4.44	10	8	19.038222	65.1706	48
16.5	0.51	0.65	1.85	4.39	4	8	26.755737	74.4686	56

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
17	0.75	0.81	1.22	5.42	20	14	25.549844	76.7774	58
17.5	0.61	0.87	1.19	6.05	17	20	9.601315	95.0817	74
18	0.43	0.8	1.14	7.59	24	20	25.073927	101.8794	80
18.5	0.8	0.7	1.41	4.62	9	8	7.619522	83.6753	64
19	0.94	0.45	1.24	4.54	21	30	18.086486	100.7488	79
19.5	0.8	0.96	1.12	6.54	18	14	5.732007	83.6753	64
20	0.97	0.41	0.82	6.75	14	18	16.754669	98.4846	77
21	0.62	0.63	1.88	4.95	21	24	13.867794	106.3946	84
22	0.62	0.63	0.56	5.42	17	20	26.043592	90.5312	70
23	0.8	0.58	1.32	4.41	30	20	29.087503	103.0096	81
24	0.62	0.9	0.8	3.79	17	30	13.591643	95.0817	74
25	0.41	0.62	0.81	5.41	25	18	31.970864	97.3509	76
26	0.86	0.91	1.51	5.43	18	22	9.473192	93.9451	73
27	0.59	0.87	1.33	4.01	32	24	20.982993	118.7509	95
28	0.43	0.72	1.46	4.66	20	28	12.742472	122.1071	98
29	0.52	0.4	1.64	4.68	15	15	12.526717	104.1389	82
30	0.95	0.83	2.23	4.84	40	10	14.812363	126.5743	102
32	0.79	0.85	0.3	3.42	25	31	10.518486	110.8979	88
34	0.9	0.49	1.7	4.14	42	14	47.843855	119.8704	96
36	0.89	0.96	0.97	3.94	28	40	10.065093	119.8704	96
38	0.52	1	1.8	4.2	38	20	9.617337	143.2469	117
40	0.43	0.44	1.43	2.42	34	40	26.768258	133.2573	108

APPENDIX G. GOODNESS OF FIT TEST RESULTS FOR NET INVENTORY WITH INSTANTANEOUS REPAIR ASSESSMENT - CONVOLUTION APPROXIMATION

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
1	0.56	0.86	0.9	8.21	1	1	12.761395	22.3621	13
1.2	0.93	0.97	2.03	3.8	2	2	14.084955	16.919	9
1.4	0.57	0.88	1.23	7.18	3	3	18.074717	24.9958	15
1.6	0.4	0.86	1.21	5.7	3	4	21.024362	31.4104	20
1.8	0.46	0.91	0.98	4.07	4	7	11.444031	30.1435	19
2	0.7	0.62	0.7	4.55	3	2	8.504929	28.8693	18
2.2	0.55	1	2.3	8.04	4	4	12.622673	40.1133	27
2.4	0.55	0.94	0.99	7.43	3	6	25.873481	32.6705	21
2.6	0.67	0.65	0.6	3.99	2	6	14.631932	28.8693	18
2.8	0.81	0.73	2.08	7.83	4	8	17.534952	40.1133	27
3	0.72	0.94	0.98	4.05	4	7	7.619581	33.9244	22
3.2	0.77	0.68	1.79	7.61	4	10	23.695209	44.9854	31
3.4	0.54	0.48	2.26	5.04	5	10	16.042827	43.7729	30
3.6	0.74	0.68	1.51	6.61	7	8	40.4659	46.1942	32
3.8	0.77	0.93	1.21	4.28	6	9	6.986432	41.3372	28
4	0.64	0.42	1.17	5.04	8	8	21.608591	43.7729	30
4.2	0.83	0.44	0.54	6.11	4	6	10.867147	42.5569	29
4.4	0.65	0.94	2.27	4.57	7	7	17.302609	49.9854	31
4.6	0.91	0.9	1.11	7.5	6	15	21.098001	44.9854	31
4.8	0.9	0.98	0.38	7.94	5	20	9.657205	47.3998	33
5	0.67	0.51	0.31	5.25	5	12	12.253753	49.8019	35
5.2	0.93	0.6	2.06	7.71	7	24	18.665303	74.4686	56
5.4	0.76	0.6	0.51	4.61	8	10	19.559415	52.1925	37
5.6	0.7	0.5	2.24	5.57	6	6	14.471527	48.6023	34
5.8	0.82	0.98	0.53	7.09	7	6	7.403263	43.7729	30

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
6	0.73	0.94	1.58	6.76	8	14	29.201553	52.1925	37
6.2	0.41	0.93	1.81	6.08	8	8	10.513709	54.5724	39
6.4	0.79	0.48	1.61	6.09	10	8	25.625971	54.5724	39
6.6	0.97	0.44	1.37	6.01	5	24	11.87001	73.3115	55
6.8	0.88	0.66	0.52	8.26	4	10	25.305109	53.3837	38
7	0.5	0.8	2	4.85	6	18	12.692191	68.6693	51
7.2	0.82	0.77	1.79	5.02	12	22	17.569716	67.5048	50
7.4	0.81	0.89	0.36	7.92	9	12	13.662902	54.5724	39
7.6	0.47	0.58	0.38	8.2	10	8	36.730349	62.8295	46
7.8	0.79	0.81	0.73	8.41	7	10	12.624371	55.7585	40
8	0.53	0.43	2.17	6.12	8	20	13.306837	70.9934	53
8.2	0.8	0.93	2.06	7.9	13	15	7.199732	70.9934	53
8.4	0.96	0.67	0.66	5.96	4	4	19.814451	41.3372	28
8.6	0.4	0.74	1.55	5.74	9	12	17.827753	66.3386	49
8.8	0.54	0.6	0.52	3.51	10	9	19.497382	56.9424	41
9	0.68	0.6	1	6.56	5	10	17.319505	60.4808	44
9.5	0.78	0.84	1.06	7.25	11	20	12.117841	73.3115	55
10	0.73	0.61	1.52	4.9	8	5	20.802873	59.3034	43
10.5	0.81	0.9	1.63	4.16	6	10	8.142263	60.4808	44
11	0.56	0.76	0.5	4.4	12	10	22.415481	74.4686	56
11.5	0.64	0.44	0.9	3.85	13	17	18.5476	73.3115	55
12	0.43	0.81	0.94	3.64	14	9	14.45552	67.5048	50
12.5	0.9	0.69	1.25	7.78	9	12	14.012025	73.3115	55
13	0.75	0.45	1.25	5.43	5	5	20.416773	73.3115	55
13.5	0.56	0.57	0.43	7.2	10	15	29.744751	74.4686	56
14	0.66	0.63	0.36	6.48	14	11	46.565626	74.4686	56
14.5	0.81	0.81	2.09	6.37	11	17	9.57954	88.2501	68
15	0.97	0.56	1.8	5.57	15	10	17.701265	73.3115	55
15.5	0.44	0.58	0.88	5.43	17	20	20.285973	92.8085	72
16	0.78	0.82	0.64	4.44	10	8	14.918159	65.1706	48
16.5	0.51	0.65	1.85	4.39	4	8	23.605815	74.4686	56

D	CRR	RSR	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
17	0.75	0.81	1.22	5.42	20	14	25.359546	76.7774	58
17.5	0.61	0.87	1.19	6.05	17	20	8.820004	95.0817	74
18	0.43	0.8	1.14	7.59	24	20	24.158464	101.8794	80
18.5	0.8	0.7	1.41	4.62	9	8	5.935937	83.6753	64
19	0.94	0.45	1.24	4.54	21	30	13.709783	100.7488	79
19.5	0.8	0.96	1.12	6.54	18	14	4.364546	83.6753	64
20	0.97	0.41	0.82	6.75	14	18	15.812853	98.4846	77
21	0.62	0.63	1.88	4.95	21	24	13.889349	106.3946	84
22	0.62	0.63	0.56	5.42	17	20	25.2205	90.5312	70
23	0.8	0.58	1.32	4.41	30	20	31.184938	103.0096	81
24	0.62	0.9	0.8	3.79	17	30	8.54167	95.0817	74
25	0.41	0.62	0.81	5.41	25	18	31.170908	97.3509	76
26	0.86	0.91	1.51	5.43	18	22	7.073874	93.9451	73
27	0.59	0.87	1.33	4.01	32	24	20.479228	118.7509	95
28	0.43	0.72	1.46	4.66	20	28	11.875581	122.1071	98
29	0.52	0.4	1.64	4.68	15	15	13.243124	104.1389	82
30	0.95	0.83	2.23	4.84	40	10	17.303654	126.5743	102
32	0.79	0.85	0.3	3.42	25	31	5.580264	110.8979	88
34	0.9	0.49	1.7	4.14	42	14	59.112436	119.8704	96
36	0.89	0.96	0.97	3.94	28	40	3.616775	119.8704	96
38	0.52	1	1.8	4.2	38	20	5.451792	143.2469	117
40	0.43	0.44	1.43	2.42	34	40	28.236857	133.2573	108

APPENDIX H. GOODNESS OF FIT TEST RESULTS FOR INVENTORY POSITION WITH NON-INSTANTANEOUS REPAIR ASSESSMENT

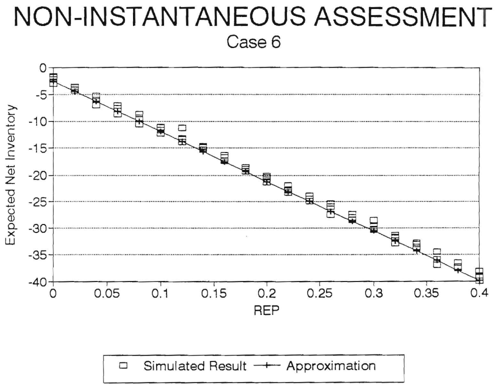
D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
1	0.56	0.86	0.05	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.05	0.7	4.55	3	2	1.832042	7.81473	3
3	0.72	0.94	0.05	0.98	4.05	4	7	6.339916	16.919	9
4	0.64	0.42	0.05	1.17	5.04	8	8	5.942194	23.6848	14
6	0.73	0.94	0.05	1.58	6.76	8	14	4.979643	31.4104	20
8	0.53	0.43	0.05	2.17	6.12	8	20	3.203709	38.8852	26
12	0.43	0.81	0.05	0.94	3.64	14	9	1.7425	32.6705	21
16	0.43	0.8	0.05	1.14	7.59	24	20	2.967227	58.1241	42
20	0.86	0.91	0.05	1.51	5.43	18	22	2.058203	53.3837	38
1	0.56	0.86	0.1	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.1	0.7	4.55	3	2	0.205652	7.81473	3
3	0.72	0.94	0.1	0.98	4.05	4	7	5.178484	16.919	9
4	0.64	0.42	0.1	1.17	5.04	8	8	4.478211	23.6848	14
6	0.73	0.94	0.1	1.58	6.76	8	14	2.832181	31.4104	20
8	0.53	0.43	0.1	2.17	6.12	8	20	3.506728	38.8852	26
12	0.43	0.81	0.1	0.94	3.64	14	9	2.99248	32.6705	21
16	0.43	0.8	0.1	1.14	7.59	24	20	3.66927	58.1241	42
20	0.86	0.91	0.1	1.51	5.43	18	22	1.843208	53.3837	38
1	0.56	0.86	0.15	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.15	0.7	4.55	3	2	0.404025	7.81473	3
3	0.72	0.94	0.15	0.98	4.05	4	7	3.354933	16.919	9
4	0.64	0.42	0.15	1.17	5.04	8	8	4.789292	23.6848	14
6	0.73	0.94	0.15	1.58	6.76	8	14	3.338997	31.4104	20
8	0.53	0.43	0.15	2.17	6.12	8	20	5.382016	38.8852	26

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
12	0.43	0.81	0.15	0.94	3.64	14	9	1.979744	32.6705	21
16	0.43	0.8	0.15	1.14	7.59	24	20	2.256624	58.1241	42
20	0.86	0.91	0.15	1.51	5.43	18	22	4.922675	53.3837	38
1	0.56	0.86	0.2	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.2	0.7	4.55	3	2	5.805768	7.81473	3
3	0.72	0.94	0.2	0.98	4.05	4	7	2.861905	16.919	9
4	0.64	0.42	0.2	1.17	5.04	8	8	5.957392	23.6848	14
6	0.73	0.94	0.2	1.58	6.76	8	14	6.055777	31.4104	20
8	0.53	0.43	0.2	2.17	6.12	8	20	6.969856	38.8852	26
12	0.43	0.81	0.2	0.94	3.64	14	9	1.934596	32.6705	21
16	0.43	0.8	0.2	1.14	7.59	24	20	1.679151	58.1241	42
20	0.86	0.91	0.2	1.51	5.43	18	22	2.412212	53.3837	38
1	0.56	0.86	0.25	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.25	0.7	4.55	3	2	0.510344	7.81473	3
3	0.72	0.94	0.25	0.98	4.05	4	7	6.473751	16.919	9
4	0.64	0.42	0.25	1.17	5.04	8	8	8.723465	23.6848	14
6	0.73	0.94	0.25	1.58	6.76	8	14	3.599857	31.4104	20
8	0.53	0.43	0.25	2.17	6.12	8	20	3.706687	38.8852	26
12	0.43	0.81	0.25	0.94	3.64	14	9	0.90591	32.6705	21
16	0.43	0.8	0.25	1.14	7.59	24	20	4.858515	58.1241	42
20	0.86	0.91	0.25	1.51	5.43	18	22	3.727339	53.3837	38
1	0.56	0.86	0.3	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.3	0.7	4.55	3	2	1.889115	7.81473	3
3	0.72	0.94	0.3	0.98	4.05	4	7	5.049341	16.919	9
4	0.64	0.42	0.3	1.17	5.04	8	8	6.562864	23.6848	14
6	0.73	0.94	0.3	1.58	6.76	8	14	3.671171	31.4104	20
8	0.53	0.43	0.3	2.17	6.12	8	20	2.354715	38.8852	26
12	0.43	0.81	0.3	0.94	3.64	14	9	1.228884	32.6705	21

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
16	0.43	0.8	0.3	1.14	7.59	24	20	3.925971	58.1241	42
20	0.86	0.91	0.3	1.51	5.43	18	22	2.382291	53.3837	38
1	0.56	0.86	0.35	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.35	0.7	4.55	3	2	1.096249	7.81473	3
3	0.72	0.94	0.35	0.98	4.05	4	7	7.281503	16.919	9
4	0.64	0.42	0.35	1.17	5.04	8	8	1.734963	23.6848	14
6	0.73	0.94	0.35	1.58	6.76	8	14	2.475652	31.4104	20
8	0.53	0.43	0.35	2.17	6.12	8	20	3.368228	38.8852	26
12	0.43	0.81	0.35	0.94	3.64	14	9	0.991155	32.6705	21
16	0.43	0.8	0.35	1.14	7.59	24	20	4.106576	58.1241	42
20	0.86	0.91	0.35	1.51	5.43	18	22	1.881931	53.3837	38
1	0.56	0.86	0.4	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.4	0.7	4.55	3	2	4.281528	7.81473	3
3	0.72	0.94	0.4	0.98	4.05	4	7	4.640284	16.919	9
4	0.64	0.42	0.4	1.17	5.04	8	8	7.842354	23.6848	14
6	0.73	0.94	0.4	1.58	6.76	8	14	2.59484	31.4104	20
8	0.53	0.43	0.4	2.17	6.12	8	20	3.635161	38.8852	26
12	0.43	0.81	0.4	0.94	3.64	14	9	3.462594	32.6705	21
16	0.43	0.8	0.4	1.14	7.59	24	20	3.809132	58.1241	42
20	0.86	0.91	0.4	1.51	5.43	18	22	1.928259	53.3837	38
1	0.56	0.86	0.45	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.45	0.7	4.55	3	2	5.954732	7.81473	3
3	0.72	0.94	0.45	0.98	4.05	4	7	1.251529	16.919	9
4	0.64	0.42	0.45	1.17	5.04	8	8	7.027937	23.6848	14
6	0.73	0.94	0.45	1.58	6.76	8	14	3.639587	31.4104	20
8	0.53	0.43	0.45	2.17	6.12	8	20	3.639694	38.8852	26
12	0.43	0.81	0.45	0.94	3.64	14	9	2.191654	32.6705	21
16	0.43	0.8	0.45	1.14	7.59	24	20	3.262054	58.1241	42

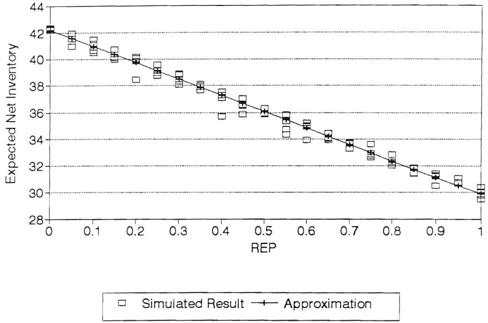
D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
20	0.86	0.91	0.45	1.51	5.43	18	22	2.322416	53.3837	38
1	0.56	0.86	0.5	0.9	8.21	1	1	0	0	0
2	0.7	0.62	0.5	0.7	4.55	3	2	3.163667	7.81473	3
3	0.72	0.94	0.5	0.98	4.05	4	7	1.325595	16.919	9
4	0.64	0.42	0.5	1.17	5.04	8	8	3.925684	23.6848	14
6	0.73	0.94	0.5	1.58	6.76	8	14	4.883142	31.4104	20
8	0.53	0.43	0.5	2.17	6.12	8	20	6.029248	38.8852	26
12	0.43	0.81	0.5	0.94	3.64	14	9	2.067457	32.6705	21
16	0.43	0.8	0.5	1.14	7.59	24	20	4.310271	58.1241	42
20	0.86	0.91	0.5	1.51	5.43	18	22	2.825833	53.3837	38

APPENDIX I. COMPARISON OF POINT ESTIMATES FOR EXPECTED NET INVENTORY WITH NON-INSTANTANEOUS REPAIR ASSESSMENT



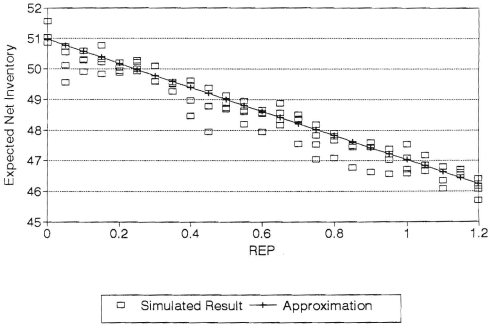
NON-INSTANTANEOUS ASSESSMENT

Case 7



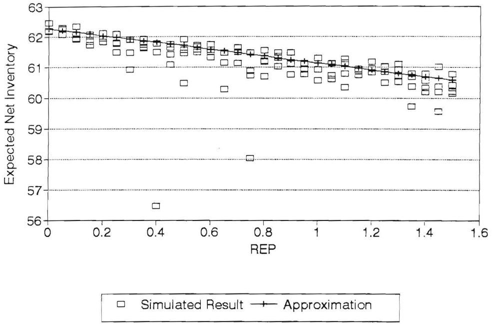
NON-INSTANTANEOUS ASSESSMENT

Case 8



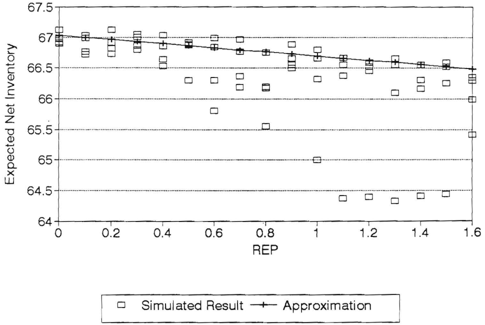
NON-INSTANTANEOUS ASSESSMENT

Case 9

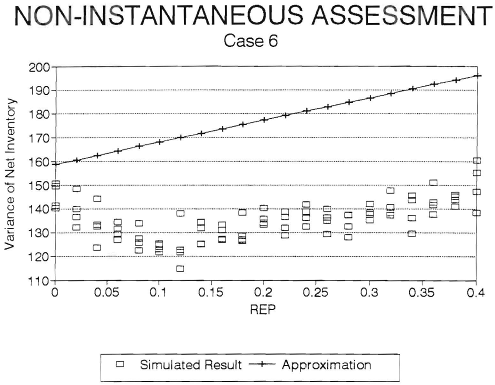


NON-INSTANTANEOUS ASSESSMENT

Case 10

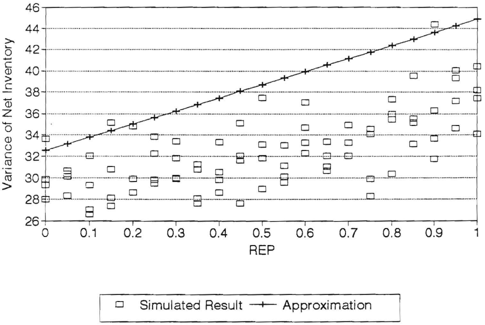


APPENDIX J. COMPARISON OF POINT ESTIMATES FOR VARIANCE OF NET INVENTORY WITH NON-INSTANTANEOUS REPAIR ASSESSMENT



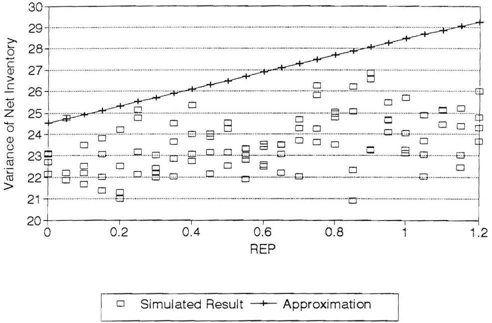
NON-INSTANTANEOUS ASSESSMENT

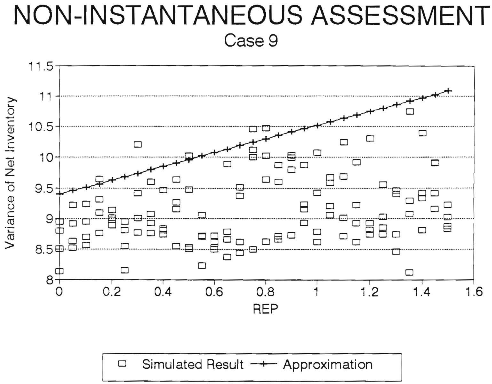
Case 7

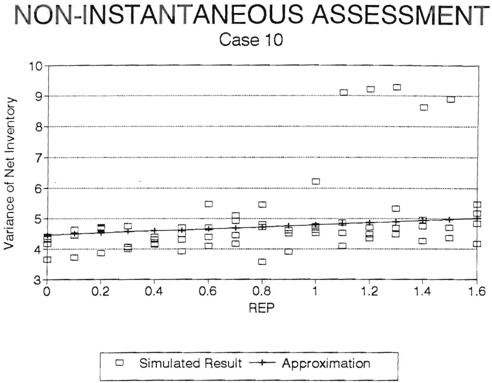


NON-INSTANTANEOUS ASSESSMENT

Case 8




NON-INSTANTANEOUS ASSESSMENT
Case 9
Variance of Net Inventory
11.5
11
10.5
10
9.5
9
8.5
8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
REP
Simulated Result
Approximation


NON-INSTANTANEOUS ASSESSMENT
Case 10
Variance of Net Inventory
10
9
8
7
6
5
4
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
REP
Simulated Result
Approximation

APPENDIX K. GOODNESS OF FIT TEST RESULTS FOR NET INVENTORY WITH NON-INSTANTANEOUS REPAIR ASSESSMENT - NORMAL APPROXIMATION

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
1	0.56	0.86	0.05	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.05	0.7	4.55	3	2	23.792937	28.8693	18
3	0.72	0.94	0.05	0.98	4.05	4	7	8.536619	33.9244	22
4	0.64	0.42	0.05	1.17	5.04	8	8	28.44658	47.3998	33
6	0.73	0.94	0.05	1.58	6.76	8	14	24.441525	52.1925	37
8	0.53	0.43	0.05	2.17	6.12	8	20	16.896291	70.9934	53
12	0.43	0.81	0.05	0.94	3.64	14	9	13.487128	66.3386	49
16	0.43	0.8	0.05	1.14	7.59	24	20	42.759145	113.1449	90
20	0.86	0.91	0.05	1.51	5.43	18	22	42.027846	75.6238	57
1	0.56	0.86	0.1	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.1	0.7	4.55	3	2	30.89962	26.2962	16
3	0.72	0.94	0.1	0.98	4.05	4	7	24.814517	37.6525	25
4	0.64	0.42	0.1	1.17	5.04	8	8	17.448078	46.1942	32
6	0.73	0.94	0.1	1.58	6.76	8	14	27.785375	52.1925	37
8	0.53	0.43	0.1	2.17	6.12	8	20	15.899915	70.9934	53
12	0.43	0.81	0.1	0.94	3.64	14	9	27.586662	65.1706	48
16	0.43	0.8	0.1	1.14	7.59	24	20	62.235448	90.5312	70
20	0.86	0.91	0.1	1.51	5.43	18	22	43.938006	76.7774	58
1	0.56	0.86	0.15	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.15	0.7	4.55	3	2	28.671046	26.2962	16
3	0.72	0.94	0.15	0.98	4.05	4	7	14.69815	35.1725	23
4	0.64	0.42	0.15	1.17	5.04	8	8	24.651764	46.1942	32
6	0.73	0.94	0.15	1.58	6.76	8	14	26.460645	54.5724	39
8	0.53	0.43	0.15	2.17	6.12	8	20	21.857007	75.6238	57

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
12	0.43	0.81	0.15	0.94	3.64	14	9	56.740418	62.8295	46
16	0.43	0.8	0.15	1.14	7.59	24	20	59.700231	89.3914	69
20	0.86	0.91	0.15	1.51	5.43	18	22	46.107932	83.6753	64
1	0.56	0.86	0.2	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.2	0.7	4.55	3	2	34.563063	26.2962	16
3	0.72	0.94	0.2	0.98	4.05	4	7	13.202221	33.9244	22
4	0.64	0.42	0.2	1.17	5.04	8	8	11.055196	43.7729	30
6	0.73	0.94	0.2	1.58	6.76	8	14	19.69969	54.5724	39
8	0.53	0.43	0.2	2.17	6.12	8	20	16.049955	74.4686	56
12	0.43	0.81	0.2	0.94	3.64	14	9	54.594807	64.001	47
16	0.43	0.8	0.2	1.14	7.59	24	20	74.159976	92.8085	72
20	0.86	0.91	0.2	1.51	5.43	18	22	40.929129	85.965	66
1	0.56	0.86	0.25	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.25	0.7	4.55	3	2	38.190173	27.5871	17
3	0.72	0.94	0.25	0.98	4.05	4	7	14.2856	31.4104	20
4	0.64	0.42	0.25	1.17	5.04	8	8	18.940755	47.3998	33
6	0.73	0.94	0.25	1.58	6.76	8	14	26.091361	55.7585	40
8	0.53	0.43	0.25	2.17	6.12	8	20	20.328911	75.6238	57
12	0.43	0.81	0.25	0.94	3.64	14	9	31.571065	64.001	47
16	0.43	0.8	0.25	1.14	7.59	24	20	56.113479	96.2165	75
20	0.86	0.91	0.25	1.51	5.43	18	22	74.727607	85.965	66
1	0.56	0.86	0.3	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.3	0.7	4.55	3	2	30.110515	28.8693	18
3	0.72	0.94	0.3	0.98	4.05	4	7	17.677939	36.4151	24
4	0.64	0.42	0.3	1.17	5.04	8	8	16.45589	46.1942	32
6	0.73	0.94	0.3	1.58	6.76	8	14	29.336062	53.3837	38
8	0.53	0.43	0.3	2.17	6.12	8	20	25.128773	75.6238	57
12	0.43	0.81	0.3	0.94	3.64	14	9	44.022946	66.3386	49

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
16	0.43	0.8	0.3	1.14	7.59	24	20	62.820411	87.1085	67
20	0.86	0.91	0.3	1.51	5.43	18	22	49.341552	92.8085	72
1	0.56	0.86	0.35	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.35	0.7	4.55	3	2	23.217909	28.8693	18
3	0.72	0.94	0.35	0.98	4.05	4	7	19.765766	33.9244	22
4	0.64	0.42	0.35	1.17	5.04	8	8	33.311977	48.6023	34
6	0.73	0.94	0.35	1.58	6.76	8	14	29.550744	50.9985	36
8	0.53	0.43	0.35	2.17	6.12	8	20	20.944245	74.7686	56
12	0.43	0.81	0.35	0.94	3.64	14	9	53.593685	60.4808	44
16	0.43	0.8	0.35	1.14	7.59	24	20	72.496857	99.6172	78
20	0.86	0.91	0.35	1.51	5.43	18	22	35.360588	90.5312	70
1	0.56	0.86	0.4	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.4	0.7	4.55	3	2	47.412649	28.8693	18
3	0.72	0.94	0.4	0.98	4.05	4	7	15.147961	33.9244	22
4	0.64	0.42	0.4	1.17	5.04	8	8	25.244493	44.9854	31
6	0.73	0.94	0.4	1.58	6.76	8	14	46.422223	59.3034	43
8	0.53	0.43	0.4	2.17	6.12	8	20	17.039916	74.4686	56
12	0.43	0.81	0.4	0.94	3.64	14	9	53.222919	66.3386	49
16	0.43	0.8	0.4	1.14	7.59	24	20	38.533070	104.1389	82
20	0.86	0.91	0.4	1.51	5.43	18	22	64.796294	91.6698	71
1	0.56	0.86	0.45	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.45	0.7	4.55	3	2	46.988943	27.5871	17
3	0.72	0.94	0.45	0.98	4.05	4	7	23.728526	31.4104	20
4	0.64	0.42	0.45	1.17	5.04	8	8	35.450828	46.1942	32
6	0.73	0.94	0.45	1.58	6.76	8	14	31.501537	52.1925	37
8	0.53	0.43	0.45	2.17	6.12	8	20	19.94953	72.1534	54
12	0.43	0.81	0.45	0.94	3.64	14	9	36.616339	66.3386	49
16	0.43	0.8	0.45	1.14	7.59	24	20	63.35843	98.4846	77

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
20	0.86	0.91	0.45	1.51	5.43	18	22	54.854438	98.4846	77
1	0.56	0.86	0.5	0.9	8.21	1	1	36.437592	22.3621	13
2	0.7	0.62	0.5	0.7	4.55	3	2	33.033824	27.5871	17
3	0.72	0.94	0.5	0.98	4.05	4	7	19.706908	32.6705	21
4	0.64	0.42	0.5	1.17	5.04	8	8	45.599139	52.1925	37
6	0.73	0.94	0.5	1.58	6.76	8	14	28.824348	58.1241	42
8	0.53	0.43	0.5	2.17	6.12	8	20	23.655007	72.1534	54
12	0.43	0.81	0.5	0.94	3.64	14	9	33.731189	74.4686	56
16	0.43	0.8	0.5	1.14	7.59	24	20	78.823562	99.6172	78
20	0.86	0.91	0.5	1.51	5.43	18	22	37.26795	93.9451	73

APPENDIX L. GOODNESS OF FIT TEST RESULTS FOR NET INVENTORY WITH NON-INSTANTANEOUS REPAIR ASSESSMENT - CONVOLUTION APPROXIMATION

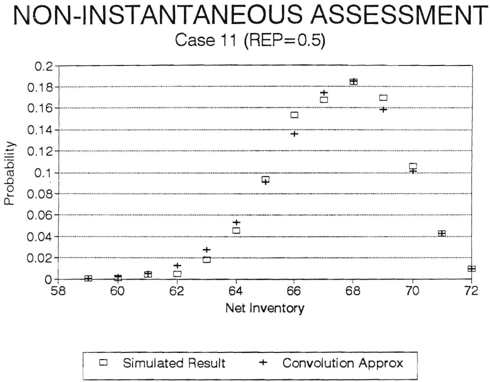
D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
1	0.56	0.86	0.05	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.05	0.7	4.55	3	2	10.580141	28.8693	18
3	0.72	0.94	0.05	0.98	4.05	4	7	8.990693	33.9244	22
4	0.64	0.42	0.05	1.17	5.04	8	8	31.889717	47.3998	33
6	0.73	0.94	0.05	1.58	6.76	8	14	26.063501	52.1925	37
8	0.53	0.43	0.05	2.17	6.12	8	20	12.659687	70.9934	53
12	0.43	0.81	0.05	0.94	3.64	14	9	13.428029	66.3386	49
16	0.43	0.8	0.05	1.14	7.59	24	20	44.022047	113.1449	90
20	0.86	0.91	0.05	1.51	5.43	18	22	43.118399	75.6238	57
1	0.56	0.86	0.1	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.1	0.7	4.55	3	2	6.87995	26.2962	16
3	0.72	0.94	0.1	0.98	4.05	4	7	13.431709	37.6525	25
4	0.64	0.42	0.1	1.17	5.04	8	8	15.213948	46.1942	32
6	0.73	0.94	0.1	1.58	6.76	8	14	26.134285	52.1925	37
8	0.53	0.43	0.1	2.17	6.12	8	20	14.543465	70.9934	53
12	0.43	0.81	0.1	0.94	3.64	14	9	27.68042	65.1706	48
16	0.43	0.8	0.1	1.14	7.59	24	20	63.3203	90.5312	70
20	0.86	0.91	0.1	1.51	5.43	18	22	45.482399	76.7774	58
1	0.56	0.86	0.15	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.15	0.7	4.55	3	2	12.08988	26.2962	16
3	0.72	0.94	0.15	0.98	4.05	4	7	9.52721	35.1725	23
4	0.64	0.42	0.15	1.17	5.04	8	8	24.017196	46.1942	32
6	0.73	0.94	0.15	1.58	6.76	8	14	24.261553	54.5724	39

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
8	0.53	0.43	0.15	2.17	6.12	8	20	18.470450	75.6238	57
12	0.43	0.81	0.15	0.94	3.64	14	9	58.833788	62.8295	46
16	0.43	0.8	0.15	1.14	7.59	24	20	61.894064	89.3914	69
20	0.86	0.91	0.15	1.51	5.43	18	22	46.325528	83.6753	64
1	0.56	0.86	0.2	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.2	0.7	4.55	3	2	18.375656	26.2962	16
3	0.72	0.94	0.2	0.98	4.05	4	7	10.23301	33.9244	22
4	0.64	0.42	0.2	1.17	5.04	8	8	10.547528	43.7729	30
6	0.73	0.94	0.2	1.58	6.76	8	14	20.503785	54.5724	39
8	0.53	0.43	0.2	2.17	6.12	8	20	13.474367	74.4686	56
12	0.43	0.81	0.2	0.94	3.64	14	9	56.549991	64.001	47
16	0.43	0.8	0.2	1.14	7.59	24	20	76.849591	92.8085	72
20	0.86	0.91	0.2	1.51	5.43	18	22	43.470878	85.965	66
1	0.56	0.86	0.25	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.25	0.7	4.55	3	2	17.359506	27.5871	17
3	0.72	0.94	0.25	0.98	4.05	4	7	13.566534	31.4104	20
4	0.64	0.42	0.25	1.17	5.04	8	8	17.222188	47.3998	33
6	0.73	0.94	0.25	1.58	6.76	8	14	26.450084	55.7585	40
8	0.53	0.43	0.25	2.17	6.12	8	20	17.1872	75.6238	57
12	0.43	0.81	0.25	0.94	3.64	14	9	31.44386	64.001	47
16	0.43	0.8	0.25	1.14	7.59	24	20	58.645602	96.2165	75
20	0.86	0.91	0.25	1.51	5.43	18	22	75.908783	85.965	66
1	0.56	0.86	0.3	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.3	0.7	4.55	3	2	13.592367	28.8693	18
3	0.72	0.94	0.3	0.98	4.05	4	7	22.758767	36.4151	24
4	0.64	0.42	0.3	1.17	5.04	8	8	17.64473	46.1942	32
6	0.73	0.94	0.3	1.58	6.76	8	14	30.308881	53.3837	38
8	0.53	0.43	0.3	2.17	6.12	8	20	25.354043	75.6238	57

D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\upsilon}$	υ
12	0.43	0.81	0.3	0.94	3.64	14	9	44.526958	66.3386	49
16	0.43	0.8	0.3	1.14	7.59	24	20	65.238899	87.1085	67
20	0.86	0.91	0.3	1.51	5.43	18	22	52.494335	92.8085	72
1	0.56	0.86	0.35	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.35	0.7	4.55	3	2	7.451515	28.8693	18
3	0.72	0.94	0.35	0.98	4.05	4	7	15.638179	33.9244	22
4	0.64	0.42	0.35	1.17	5.04	8	8	29.616527	48.6023	34
6	0.73	0.94	0.35	1.58	6.76	8	14	25.145548	50.9985	36
8	0.53	0.43	0.35	2.17	6.12	8	20	20.012649	74.7686	56
12	0.43	0.81	0.35	0.94	3.64	14	9	55.218585	60.4808	44
16	0.43	0.8	0.35	1.14	7.59	24	20	75.688889	99.6172	78
20	0.86	0.91	0.35	1.51	5.43	18	22	37.066806	90.5312	70
1	0.56	0.86	0.4	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.4	0.7	4.55	3	2	16.747985	28.8693	18
3	0.72	0.94	0.4	0.98	4.05	4	7	15.303724	33.9244	22
4	0.64	0.42	0.4	1.17	5.04	8	8	26.009669	44.9854	31
6	0.73	0.94	0.4	1.58	6.76	8	14	45.614462	59.3034	43
8	0.53	0.43	0.4	2.17	6.12	8	20	16.461871	74.4686	56
12	0.43	0.81	0.4	0.94	3.64	14	9	52.745219	66.3386	49
16	0.43	0.8	0.4	1.14	7.59	24	20	39.36490	104.1389	82
20	0.86	0.91	0.4	1.51	5.43	18	22	64.712694	91.6698	71
1	0.56	0.86	0.45	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.45	0.7	4.55	3	2	20.790715	27.5871	17
3	0.72	0.94	0.45	0.98	4.05	4	7	16.433011	31.4104	20
4	0.64	0.42	0.45	1.17	5.04	8	8	37.320679	46.1942	32
6	0.73	0.94	0.45	1.58	6.76	8	14	31.380892	52.1925	37
8	0.53	0.43	0.45	2.17	6.12	8	20	17.459983	72.1534	54
12	0.43	0.81	0.45	0.94	3.64	14	9	38.791877	66.3386	49

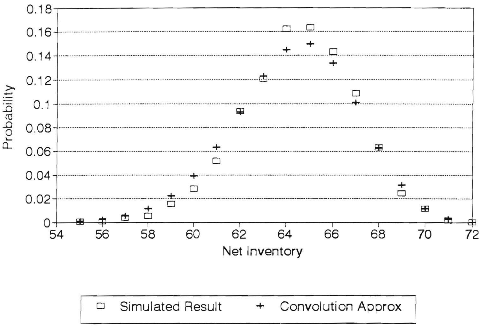
D	CRR	RSR	REP	RTAT	PCLT	Q_R	Q_P	χ^2	$\chi^2_{.95,\,\upsilon}$	υ
16	0.43	0.8	0.45	1.14	7.59	24	20	64.822505	98.4846	77
20	0.86	0.91	0.45	1.51	5.43	18	22	59.09522	98.4846	77
1	0.56	0.86	0.5	0.9	8.21	1	1	12.761395	22.3621	13
2	0.7	0.62	0.5	0.7	4.55	3	2	18.136201	27.5871	17
3	0.72	0.94	0.5	0.98	4.05	4	7	15.255755	32.6705	21
4	0.64	0.42	0.5	1.17	5.04	8	8	41.520235	52.1925	37
6	0.73	0.94	0.5	1.58	6.76	8	14	27.599098	58.1241	42
8	0.53	0.43	0.5	2.17	6.12	8	20	24.805297	72.1534	54
12	0.43	0.81	0.5	0.94	3.64	14	9	31.720841	74.4686	56
16	0.43	0.8	0.5	1.14	7.59	24	20	81.964977	99.6172	78
20	0.86	0.91	0.5	1.51	5.43	18	22	42.008403	93.9451	73

APPENDIX M. COMPARISON OF PROBABILITY DISTRIBUTIONS FOR NET INVENTORY WITH NON-INSTANTANEOUS REPAIR ASSESSMENT - CONVOLUTION APPROXIMATION



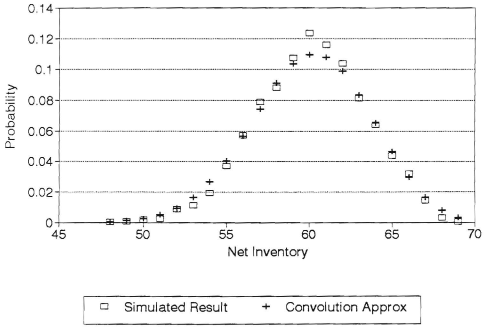
NON-INSTANTANEOUS ASSESSMENT

Case 12 (REP=0.5)



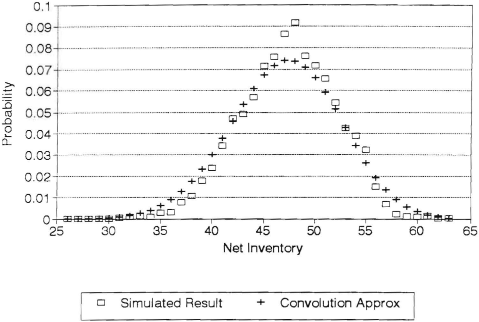
NON-INSTANTANEOUS ASSESSMENT

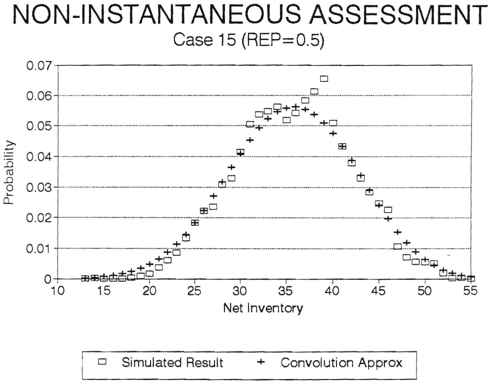
Case 13 (REP=0.5)



NON-INSTANTANEOUS ASSESSMENT

Case 14 (REP=0.5)

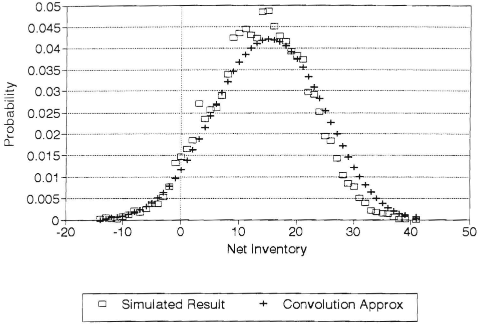


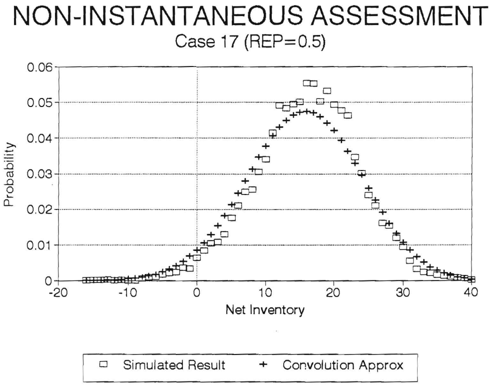


NON-INSTANTANEOUS ASSESSMENT
Case 15 (REP=0.5)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Probability
10
15
20
25
30
35
40
45
50
55
Net Inventory
Simulated Result
Convolution Approx

NON-INSTANTANEOUS ASSESSMENT

Case 16 (REP=0.5)

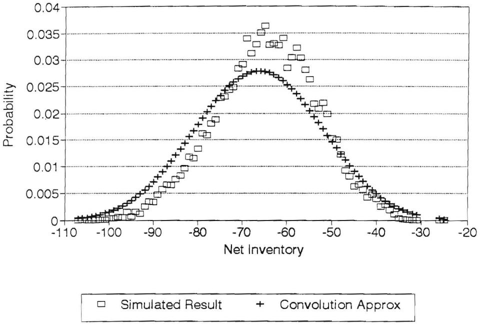


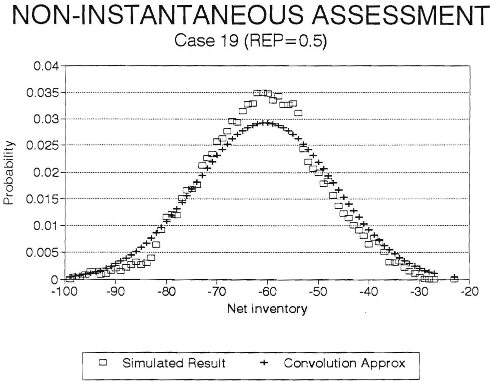


NON-INSTANTANEOUS ASSESSMENT
Case 17 (REP=0.5)
0.06
0.05
0.04
0.03
0.02
0.01
0
Probability
-20
-10
0
10
20
30
40
Net Inventory
Simulated Result
Convolution Approx

NON-INSTANTANEOUS ASSESSMENT

Case 18 (REP=0.5)




NON-INSTANTANEOUS ASSESSMENT
Case 19 (REP=0.5)
Probability
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-100
-90
-80
-70
-60
-50
-40
-30
-20
Net Inventory
Simulated Result
Convolution Approx

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